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SCHOOL SCIENCE AND MATHEMATICS

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IMPROVEMENTS IN APPEARANCE AND UTILITY

For many years SCHOOL SCIENCE AND MATHEMATICS has presented the same external appearance. At the last annual meeting of the Board of Directors a number of changes were authorized. The Journal Committee, under the leadership of Mr. Theodore Kelsey, Chairman, and ably seconded by Mr. G. E. Hawkins, recommended a change in cover design and stock, improved typography, and other alterations tending toward simplicity and greater ease in reading.

The new cover was selected from several designs submitted by Mr. Alfred Sterges of Chicago. Final work by members of the Editorial Staff, Business Manager W. F. Roecker, and our printer, the George Banta Publishing Company of Menasha, Wisconsin enables us to present the April issue in modern apparel. The purpose of the journal—to coordinate and improve the teaching of the basic sciences in all the schools—remains unchanged. This is a cooperative project in which every teacher of mathematics and science may take part and share in the benefits. We hope you will approve the changes and that we may continue to merit your confidence and support.

But now that certain mechanical improvements have been made we are by no means satisfied. The content is a matter of much greater import than the superficial appearance. This is the foundation on which SCHOOL SCIENCE AND MATHEMATICS was built and has prospered. The constant use of the current and bound volumes of this journal in the great teacher training institutions is evidence of their intrinsic worth. This is the fundamental property to be maintained and will continue to be our chief concern.

A NEW CHEMISTRY EDITOR

Mr. Druley Parker, Chemistry Editor for the past year, left the teaching profession at the end of the fall semester and relinquished his editorial work. During the few months of his service Mr. Parker was one of our most active staff members and proved his editorial ability. Many thanks, Mr. Parker, and best wishes for success in your new field.

E. G. Marshall, Ph.D., Chemistry Instructor at the LaSalle-Peru-Oglesby High School and Junior College, was appointed to succeed Mr. Parker as Chemistry Editor. Dr. Marshall holds degrees from Indiana University and the University of Chicago. After receiving his bachelor's degree he began his teaching career in the high school at Carthage, Illinois. Graduate study brought him in touch with the industrial world, where he spent a number of years as a research chemist, first in the rubber industry, then in dyes. But people interested him more than things, and he returned to teaching. For the past several years he has taught both high school and college chemistry in his present position at LaSalle. The thorough grounding in the subject matter of science received at I.U., the mother of great teachers, and at the great Midway research institution, supplemented by rich experience in both research and teaching, especially fits him for a place on our editorial board. Welcome Mr. Marshall.

A DIFFERENT ATTITUDE TOWARD WAR

An editorial entitled, "Nineteen Hundred and Forty," in the January issue of *The B. C. Teacher* is worthy of notice. We quote:

"The Editor of *The B. C. Teacher* can compete neither with Isaiah nor with Dyson and, looking ahead, he sees little light. From one glimmering taper, however, he derives comfort which he would fein share with his readers.

"The most hopeful thing evident in the whole current situation is the different attitude toward war that has become a commonplace within a generation. We may well believe that never before in the world's history has so profound and significant a spiritual revolution re-oriented the thinking of so many million people.

"Today, few indeed see in war the alluring adventure that many imagined they saw in it in 1914. We know better what war does to men; to those who die and, alas, to those who live."

METHODS AND IMPORTANCE OF pH AND ACIDITY MEASUREMENTS

MALCOLM DOLE
Associate Professor of Chemistry, Northwestern University

In the old days, I do not say the good old days, one studied chemistry because of the intellectual and aesthetic pleasure one experienced in learning of the manifold laws, principles and facts of this fascinating subject, but in these good new days in which we are now living, we study chemistry, not because of its abstract satisfaction, but because it can be related to everyday life, because it can be integrated in terms of human experience, because to it we can apply the "dynamogenic" method of teaching to use a phrase of Dr. Stevens of Northwestern University's University College. This change in our attitude toward the learning of chemistry is aptly illustrated by the following story, a true story, which rather uniquely demonstrates the difference between the Mid-Victorian or idealistic and the modern concepts of love. In Buzzards Bay, Massachusetts, there is a small island, known as Amrita Island, connected to the mainland through a viaduct and abruptly terminated by a low cliff from whose topmost rocks can be obtained a beautiful view of the Bay and, across the water, of the land stretching far to the west. On the brow of this cliff the nineteenth century owners of the island (they are long since dead) built for themselves an immense mausoleum of field stone, a tomb closed with heavy bronze doors over which one can read engraved in the metal, the words "Love is Eternal." A friend of mine as a young girl used to explain to visitors, that the tomb's inscription read "Love is Internal." Chemistry and pH, the subject of which I speak today is assuredly both "Eternal" and "Internal"!

In Samuel Johnson's[1] classical Dictionary of the English language which describes a "chymist" as a "philosopher by fire," the word "acid" is defined as follows:

> Acid. Liquors and substances are called acid which, being composed of pointed particles, affect the taste in a sharp and piercing manner. The common way of trying, whether any particular liquor hath in it any particles of this kind, is by mixing it with a syrup of violets, when it will turn of a red colour; but if it contains alkaline or lixivial particles, it changes that syrup green.

[1] *Samuel Johnson's Dictionary*, 4th ed. W. Strahan, London, 1773.

Although this definition examined in the light of modern theories of solutions does not seem to be too far wrong, it remained for Arrhenius over a hundred years later to give us our first concept of ions, indeed it was Arrhenius's study of the equivalent conductance of acids and the comparison of the conducting power of the acids with their chemical strengths that suggested his ionic theory and enabled us to identify Samuel Johnson's "pointed particles" as the hydrogen ion, or more exactly the nucleus of the hydrogen atom or the proton hydrated with one molecule of water. The hydrogen ion, an ion of about the same size as the potassium ion, is therefore a proton hydrated with just one molecule of water, an ion that can further hydrate itself in the same way that the potassium ion can also bind to itself rather indefinitely, but nevertheless most strongly, an unspecified number of water molecules.

To obtain a measure of the acidity of a solution it is insufficient as Sörensen emphatically pointed out to titrate with base, thereby finding the normality or what we might call the stoichiometric concentration of the solution; instead if we wish to find the acidity of the solution which is important in controlling the rate of certain chemical reactions, which is important in determining the properties of proteins, which is important in realizing the proper taste of beer or of Jello or in making possible the tanning process, we must determine the instantaneous concentration of the hydrogen ion existing in the solution during the time that the desired process is carried out. But how are we going to perform this difficult task? We cannot do it by measuring the conductivity of the solution or its osmotic pressure or its freezing point lowering, because all of these phenomena depend upon all of the ions in the solution; we must measure some property whose magnitude depends specifically and solely upon the hydrogen ion concentration. Unfortunately there has never been discovered any such unique property, so that in solutions such as blood, or saliva, or a tanning extract containing a complex mixture of ions, colloids and other substances we may never know exactly the hydrogen ion concentration. However the situation is not as hopeless as these remarks have indicated; we can measure the pH by a number of methods, because the pH is today defined in terms of the electromotive force or potential developed by the so-called hydrogen electrode immersed in the test solution when this solution is contained in the appropriate electrochemical cell, mathematically

$$\mathrm{pH} = F\frac{(E-E_0)}{2.3\ RT} \quad \text{or at } 25°\ \mathrm{C}, \quad \mathrm{pH} = \frac{E-E_0}{.05915} \tag{1}$$

where the constant E_0 depends on the reference electrode. Fig. 1 illustrates a simple experimental arrangement for the measurement of the pH using a hydrogen electrode, the latter consisting of platinum covered with a thin layer of catalytic platinum

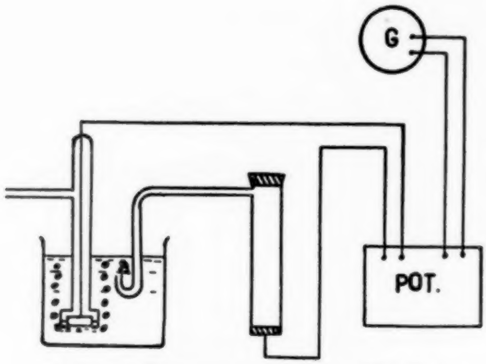

Fig. 1. Hydrogen electrode pH cell.

black deposited electrolytically which is in equilibrium with the hydrogen gas bubbling through the cell. If the reference electrode is the mercury, mercurous chloride, saturated potassium chloride solution electrode as it is in Fig. 1, then the hydrogen electrode is negative and the constant of Eq. 1 is 0.2450 volt. Although the voltage of the hydrogen electrode at constant temperature and pressure theoretically in the case of perfect or ideal solutions is a sole function of the hydrogen ion concentration, actually it is not as no solution is ideal, the potential really being a measure of the activity of the hydrogen ion. The activity function is the concentration multiplied by the activity coefficient, and as we know fairly well in general how the activity coefficient varies with the constitution of the solution, it might be thought that the use of the hydrogen electrode would after all provide us with a method of measuring hydrogen ion concentration. But here again there is an unexpected difficulty caused by the fact that in our electrochemical cell there exists at *a*, Fig. 1, a liquid junction potential which we must know in order to calculate ionic activities. However, to compute the liquid junction potential (which cannot be measured) we must have the individual ionic activities; thus we are required to know in

advance that which we wish to find and are led, therefore, into a vicious circle from which escape is only possible by abandoning our attempt to discover the exact thermodynamic relationship between hydrogen ion concentration and cell voltage. It is for this reason that we do not define pH as did Sörensen as the negative of the logarithm of the hydrogen ion concentration; instead we give to it the practical basis and operational significance illustrated by Eq. 1.

In Fig. 2 we show the connection between e.m.f., pH and normality during the titration of a weak and strong acid with the strong base sodium hydroxide. The pH of pure water is 7.00; if the pH is higher than 7 the solution is basic, if lower it is

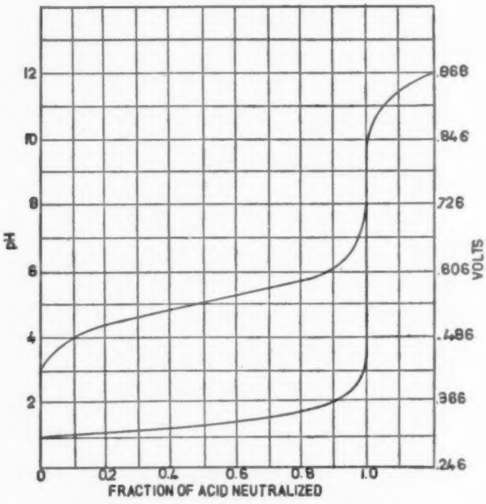

FIG. 2. Titration curves for acetic (top) and hydrochloric (bottom) acids.

acid; the pH of 8 N hydrochloric acid, for example, is about -1.17. Note the difference in the shape of the titration curves, that for the hydrochloric acid titration being typical for a completely dissociated substance while the curve for acetic acid is characteristic of an acid whose partial dissociation is governed at all concentrations by the law of mass action. Knowing the pH when the weak acid is half titrated, we can obtain immediately the dissociation constant (pK in negative logarithm units) of the acid inasmuch as the hydrogen ion concentration at this particular point is equal to the dissociation constant. (I

might add parenthetically that we adjust the constant E_0 of Eq. 1 to make this statement true.)

Although the pH is defined in terms of the e.m.f. of the hydrogen electrode cell and although the hydrogen electrode method is the standard to which all other methods of pH measurement must ultimately be referred, seldom if ever do we use this type of electrochemical cell in actual practice because of the ease with which the catalytic surface of platinum black is poisoned, the most common poison being oxygen of the air. It is inconvenient to perform measurements carefully excluding air.

Within the last ten years a remarkable electrode whose active surface is simply a glass film, has been developed and has recently found widespread application in academic research laboratories, industrial control laboratories, in hospitals and biological research institutions. I, myself, am deeply interested in this glass electrode and have devoted many years to its study. Fig. 3

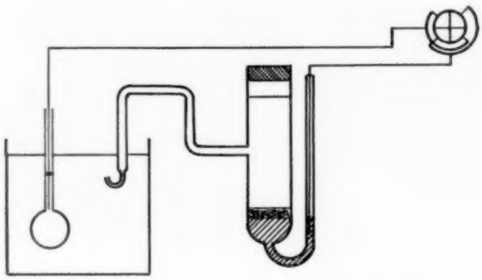

FIG. 3. Haber's glass electrode cell.

illustrates the bulb type of glass electrode originally due to Haber; on the outside surface of the glass bulb a potential is produced which depends upon the hydrogen ion concentration of the solution in exactly the same manner as does the potential of the hydrogen electrode except in alkaline and non-aqueous solutions where the glass electrode fails rather badly. The chief difficulty with the glass electrode is the enormous electrical resistance experienced by the current in trying to flow directly through the glass wall as it must during the e.m.f. measurement; for that reason Haber was forced to use a quadrant electrometer to indicate the e.m.f., an instrument which is not too easy to manipulate. Now-a-days fortunately we can use instead vacuum tube amplifiers or electrometers such as the Coleman pH electrometer illustrated in Fig. 4, an instrument which like other

similar commercial outfits can be made by fairly large scale production methods, which is rugged, self-containing and portable and which is practically fool-proof in operation. Using these industrial pH electrometers, the pH measurement is reduced to an amazing simplicity. One needs after a preliminary standardization of the electrode only to put the test sample into the little glass cup, to raise the cup up around the electrode bulb, to adjust the potentiometer knob until the ammeter needle shows no deflection on tapping the key and finally to read the pH on the pH dial. The possible error is ± 0.04 pH units, but with care and with the use of accurate research instru-

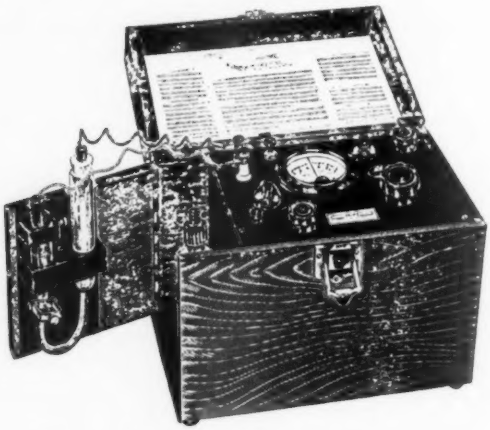

FIG. 4. A commercial pH electrometer.

ments pH measurements with the glass electrode to 0.001 pH unit have been made. No chemical need be added to the test sample, nor is it necessary to exclude oxygen. Gases, colloids, proteins or salts do not poison the electrode; it is only in error when placed in solutions more basic than pH 10, or in non-aqueous solutions or in solutions containing hydrogen fluoride whose glass corroding properties are well known. (A study of the glass electrode in fluoride solutions has just been completed by me this week.) The pH of minute quantities of solutions can be measured with the aid of micro or membrane type electrodes; colored solutions, thick, turbid colloids or gels offer no difficulties to the glass electrode.

The use of the quinhydrone electrode, of the antimony oxide electrode and of indicators in measuring the pH will not be mentioned because of lack of time; instead we shall pass im-

mediately to the subject of the importance of pH measurements.

Let us consider the study of pH in biology and medicine and here we have first of all blood, a liquid described by Goethe's Mephistopheles in the words "Blut ist ein ganz besondrer Saft." The hydrogen ion concentration of blood influences among other things the extent to which hemoglobin of the blood or the blood plasma can absorb oxygen or carbon dioxide; this is by virtue of the following equilibria which exist in the blood,

$$\begin{array}{llll}
O_2 + & H \cdot Hb \rightleftarrows H \cdot HbO_2 \rightleftarrows & H^+ + HbO_2^- & \\
& \upharpoonleft\downharpoonright & + & \\
& H^+ & HCO_3^- & \\
& + & \upharpoonleft\downharpoonright & \\
& Hb^- & H_2CO_3 \rightleftarrows H_2O + CO_2 &
\end{array}$$

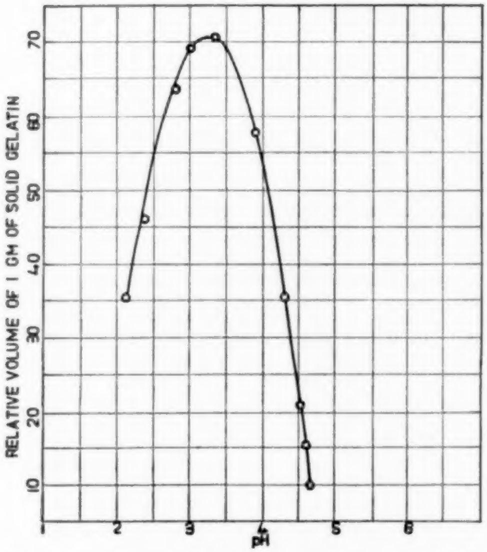

FIG. 5. Swelling of gelatin as a function of pH.

where $H \cdot Hb$ represents reduced hemoglobin and $H \cdot HbO_2$ represents oxyhemoglobin. Since $H \cdot HbO_2$ is a stronger acid than $H \cdot Hb$ and since carbonic acid is weak, increase in hydrogen ion concentration shifts the carbon dioxide equilibium to the right and downwards liberating carbon dioxide while addition of acid also liberates oxygen. The effect of increasing pressure of carbon dioxide in reducing the oxygen solubility and the analogous phenomenon of increasing oxygen tension reducing the carbon dioxide solubility is to be explained on the basis of the role the hydrogen ion plays in these equilibria.

The hydrogen ion concentration of the blood plasma likewise influences the distribution of chloride and bicarbonate ions between blood cells and blood plasma, and it is believed that the rate of respiration is determined by the pH of the blood, a slight decrease in pH produced by lactic acid formation during exercise stimulates a nerve which causes an increase in the breathing rate.

Among the most important constituents of our body we may list the various proteins, substances without which life is impossible and substances whose physical properties because of their amphoteric nature are largely determined by the pH of the

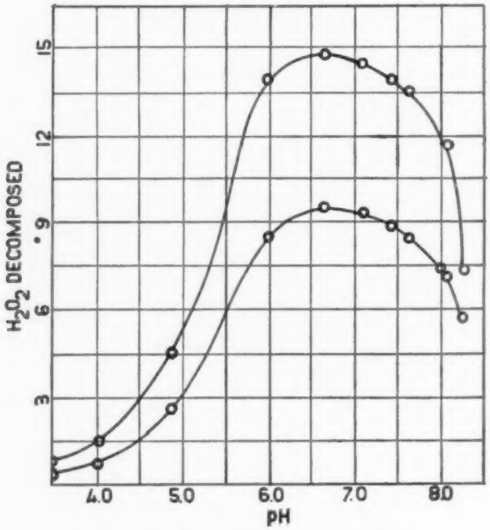

FIG. 6. Activity of enzyme catalase a function of pH.

medium in which the exist. In Fig. 5 I have plotted the swelling of the protein gelatin; relative volumes of one gram of the solid being the ordinates and the pH, the abscissas, as taken from Jacques Loeb's famous little book "Proteins and the Theory of Colloidal Behavior." The dependence on the pH is indeed startling. A similar marked influence of the pH on the activity of the proteinic enzyme catalase in catalyzing the decomposition of hydrogen peroxide is illustrated in Fig. 6; the different curves being for different reaction times. Note that a pH range exists over which the activity of the protein is a maximum.

pH measurement of the stomach juices both *in situ* and *in vitro* are now frequently made by means of the glass electrode. Dr. Osterberg of the Mayo clinic writes:[2]

> We have used the glass electrode for a research project having to do with the determination of the pH values of gastric and duodenal fluids. Our present set-up is as follows: Two-lumen tubing is swallowed, one opening remaining in the stomach and the other in the duodenum as determined by X-ray, and two tubes are kept under constant suction by means of two gastric evacuators. By means of a Y-tube glass electrodes are inserted into each circuit and the flow of duodenal or gastric content controlled by means of stopcocks. By suitable control of the stopcocks we have been able to withdraw and measure the pH of the gastric and duodenal fluids at intervals of one minute and have thus obtained accurate information as to the emptying of the stomach into the duodenum and the variation in pH values which occur following various types of test meals and other gastric and duodenal stimulants. This work has been done on normal individuals and on patients suffering with gastric or duodenal ulcers.

Using the glass electrode Dr. Fosdick of the Northwestern University Dental School has been able to measure accurately the pH of saliva and to determine the effect of the acidity of the mouth in producing or accelerating dental caries. That organic acids produced by bacteriological degradation of sugar or other carbohydrates in the mouth attack the surface enamel of the teeth is a conclusion widely accepted today. In biological research, pH measurement and control are now considered to be as important as temperature control; in addition to the previously mentioned applications, measurements have been made of the pH of the brain, of muscle tissue, of the skin, of the circulating blood, of bacteriological cultures, and of numerous body fluids and secretions.

Turning next to industrial applications, we find that chemists in the leather industry were among the first to apply the glass electrode to practical pH problems not only because the tanning of leather whether by the vegetable or mineral tanning process requires the careful attainment of the proper pH at several steps in each process, but also because the measurement of pH of colored, colloidal, and complex tanning liquors is extremely difficult if not impossible by any other method than that using the glass electrode. In vegetable tanning of hides, after a number of cleansing, hair, epidermis and fat-removing operations, the hide must experience a slight acid swelling in order that the colloidal protein collagen of skin may be made positive so that it will adsorb or possibly combine chemically with the nega-

[2] Private Communication.

tive colloid, tannin. In the case of mineral tanning, basic chromic sulfate is the tanning agent; if the chromic solution is too acid, the rate and amount of agent adsorbed by the skin is reduced to too low a value; if the chromic solution is too basic, a precipitation of the agent in the solution may take place before sufficient colloidal chromic oxide has been adsorbed by the hide. Thus in this brief account we see the importance of pH control to the leather industry and we are not surprised at the rapidity with which leather chemists have adopted and even advanced the best pH measuring techniques.

Leather is a colloid and so is rubber, particularly rubber latex whose colloidal properties and whose stability are dependent upon proper pH control. Fig. 7 demonstrates the relationship

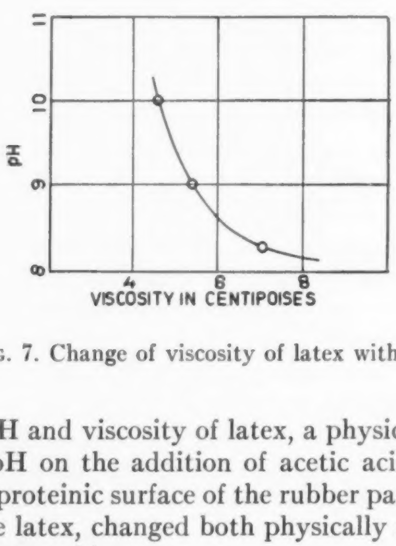

FIG. 7. Change of viscosity of latex with pH.

between the pH and viscosity of latex, a physical property; by lowering the pH on the addition of acetic acid, the negative charge on the proteinic surface of the rubber particle is brought to zero and the latex, changed both physically and chemically, coagulates to pure rubber.

Throughout the food industry food chemists make many pH measurements as part of the routine work of food manufacture and control; to illustrate the variety of substances whose hydrogen ion concentration is investigated in just one manufacturing plant, a meat packing house, I shall read to you a list kindly communicated to me by Dr. R. C. Newton, chief chemist, Swift and Company, Chicago.

Adhesives
Bacteriological Cultures
Brines (Cooling System)

- Blood Solutions
- Buffers
- Butter Serum
- Cheeses
- Cloth (Water Extract)
- Curing Pickles (Meat)
- Earth (Clay and Extract)
- Eggs (Whites and Yolks)
- Flexible Glue
- Fruit Juices
- Gelatin
- Gelatin Liquors
- Ink
- Invert Syrup
- Meat Extracts
- Meats (Fresh and Cured)
- Meat Juices
- Milk
- Paper (Water Extract)
- Reagents (Oxygen Absorbent)
- Reconstituted Powdered Milk Sewage
- Soap and Soap Powder Solutions
- Solutions for Sulfide Precipitations from Various Determinations
- Water
    - Boiler Water
    - City Water Supply
    - Wash Waters
    - Blue Gas and Hydrogen
    - Scrubber Waters

As to the significance of the pH measurements in the case of each of these materials, I cannot in general say; usually the significance is empirical, I mean by this that plant experience will teach the optimum pH for each substance or process, the trial and error method being used to obtain exact knowledge of this pH.

The pH of canned foods or of fruit juices is of significance in determining the extent of corrosion of the tin and in determining the extent of oxygen consumption and hydrogen evolution (in some cases the hydrogen pressure within the can has risen as high as two atmospheres). Since in many cases growing bacteria cause the formation of acid, a sudden drop in pH of a foodstuff may indicate that decomposition has set in; the formation of butyric acid in butter and of lactic acid in milk is an example of this. Speaking of milk I might add that the coagulation of milk to form a curd is an optimum within the pH range 5.99 to 6.40; thus pH measurement has its practical advantages in the cheese industry.

Sourness, one of the four elements of flavor, is directly related to titratable acidity and indirectly to pH, so that a pH measure-

ment is helpful in controlling the flavor of food products. I am told that the pH of Jello must be adjusted within ± 0.01 pH unit, if it is slightly too basic, it will taste flat or soapy, if too acid, its flavor will be too sharp. For gel formation it is necessary in the household manufacture of jelly that not only must the fruit juice contain pectin, but it must be sufficiently acid; as a suggestion, one can add a level teaspoon of powdered tartaric or citric acid to a quart of fruit juice; the liquor should then be as acid to taste as a good tart apple, if it is not, more powdered acid can be cautiously added. Substances not well adapted to jelly manufacture such as peach, pear, strawberry, cherry, sweet apple, and others deficient in acid can be improved in regard to the production of a gel by the addition of acid; yet too much acid should be avoided for the delicate fruit flavors may be impaired; pear, peach and strawberry jellies do not have their flavor improved by the addition of acid; sweet apple jelly does.

In conclusion may I express the thought that perhaps it will become possible for you as teachers to use the small portable glass electrode outfits in your laboratories for the purpose of the measurement of pH of common, ordinary solutions one meets every day in practical experience, such as tap water or other liquids already mentioned. The simplicity of operation of commercial pH electrometers commends them for instructional use in the realm of pH and acidity, and makes possible vivid demonstrations of variations in hydrogen ion concentration.

MINERALS OF ALASKA FOR 1939

Although the year 1939 set few new records of production, the results as a whole were good and encourage belief in the continuing high rate of output of minerals from the Territory.

As compared with 1938, there seems to have been a decrease in 1939 of approximately $3,700,000 in the value of the total production of minerals. The record for 1938, however, was exceedingly high, having been surpassed in only three years during the entire period that mining has been in progress in Alaska. Furthermore, none of the great copper mines, which in 1938 produced copper to the value of nearly $3,000,000, were in operation.

The value of gold produced by Alaskan mines in 1939 exceeded in value that for any other year in Alaskan history with the single exception of 1938.

The continued large output of platinum metals from Alaskan mines is a source of special national significance, as it goes far toward making the United States independent of foreign supplies.

In addition to gold and platinum, the most significant products of Alaskan mines in 1939 were coal, silver, lead, limerock, tin, antimony, and a small amount of copper.

A GENERAL FORMULA FOR MAGIC SQUARES OF VARIOUS ORDERS BEGINNING WITH NUMBERS DIFFERENT FROM UNITY

L. R. POSEY

Southern University, Scotlandville, Louisiana

Making magic squares in an old pastime. They were constructed in India about 1000 years before the Christian era and seem to have been introduced into Europe in the early part of the fifteenth century by Moschopulus, who lived in Constantinople. The mathematical theory of these squares was taken up by French scholars in the seventeenth century and since then it has been a favorite subject by writers in many countries. All kinds and types of magic squares have been constructed with integers in arithmetic progression. (See W. W. R. Ball's *Mathematical Recreations and Essays*, Chapter 7, tenth edition, for further details and methods of constructing magic squares.)

This paper is an attempt to extend the formula, given on page 137 in Ball's *Mathematical Recreations and Essays* for finding the sum of the numbers in the rows, columns, and diagonals of a magic square composed of consecutive integers from 1 to n^2, to a general formula giving the sum of the rows, columns, and diagonals of *any numbers in arithmetical progression* be they integers or fractions or algebraic irrational numbers (positive or negative or both) beginning at any point in the series.

To make the rule easy to follow a few simple terms must be defined. A magic square is one containing numbers such that the sums in the rows, columns, and diagonals are equal.

The magic squares used in this article are composed of an *odd number* of small squares. These small squares are called cells. The cells running from left to right parallel to the base of the square are called 'rows' and those running from top to bottom perpendicular to the base are called 'columns' and those running from the upper left hand to the lower right hand corners and from the lower left hand to the upper right hand corners of the square are called 'diagonals.'

The number of rows or columns determine the order of the square. The order is represented by n.

To fill a third order square, with consecutive integers from 1 to 9, do as follows: First, place the number 1 in the middle cell of the first row at the top of the square. Second, go out of the

upper right hand corner of this cell over the adjoining column and place the number 2 in the first cell at the bottom of this column. Third, go out of the upper right hand corner of this cell to the adjoining row above it and place the number 3 in the first cell to the left, in this row. Fourth, going out of the upper right hand corner of this cell leads into a cell already filled, hence, place the number 4 in the adjoining cell below it. Fifth, go out of the upper right hand corner of this cell along the diagonal of the square, placing the consecutive numbers 5 and 6 in the consecutive cells along the diagonal until the last cell of the first row in the upper right hand corner of the square is filled. The number 6 in this case occupies this cell. Sixth, place the number 7 below this cell. (Always place the next consecutive number below the number in the upper right hand corner of the square.) Seventh, go out of the upper right hand corner of this cell to the adjoining row above it and place the number 8 in the first cell to the left, in this row. Eighth, go out of the upper right hand corner of this cell over the adjoining column and place the number 9 in the first cell at the bottom of this column and the magic square will be completed. Observe that the direction in which the consecutive numbers are placed in the square is always parallel to the diagonal running from the lower left hand to the upper right hand corner of the square and that when a cell is filled to prevent this procedure always place the number below the cell out of which it was impossible to go, and start over again in the same direction. On reaching the cell in the upper right hand corner of the large square, place the next consecutive number below this cell and then proceed as before.

To make these rules clear, observe the way in which the numbers are placed in the cells of the figures accompanying this article, as they are read beginning with Figure 1.

8	1	6
3	5	7
4	9	2

FIG. 1

12	5	10
7	9	11
8	13	6

FIG. 2

Any square of an odd number of cells filled with consecutive integers from 1 to n^2 is defined in this article as the original square. The original square will be used as a basis for finding the

sum of the numbers in each row, column, and diagonal of any series of numbers ***in arithmetic progression.***

The general formula for finding the sum of the numbers in each row, column, and diagonal of the magic square defined above is:

$$S_a = dS + (a-d)n \tag{1}$$

where a stands for the first number in the series, d for the common difference, n for the order of the square and S the sum of the numbers in each row, column, and diagonal if the original square was filled with consecutive integers from 1 to n^2.

The formula for such a sum in the original square is:

$$S = \frac{n}{2}(n^2+1), \tag{2}$$

where n, of course, stands for the order of the magic square.

Formula (2) gives the sum of the numbers in the rows, columns, and diagonals when and only when, the series is a set of consecutive *integers* from 1 to n^2. Formula (1) gives, perhaps, the sum of the numbers in the rows, columns, and diagonals of *any set of rational* or algebraic irrational numbers in arithmetic progression. A few examples will make clear to the reader the principles set forth in this article.

Figure 2 is a square of the third order filled with consecutive integers from 5 to 13 in which S would be 15 as explained above. Substituting in the general formula (1) we have $S_5 = 15 + (5-1)3 = 27$, the sum of the numbers in each row, column, and diagonal of the square.

Figure 3 is a square of the third order filled with numbers

26	5	20
11	17	23
14	29	8

FIG. 3

11	-3	7
1	5	9
3	13	-1

FIG. 4

whose common difference is 3. $S_5 = 3 \times 15 + (5-3)3 = 45 + 6 = 51$, the sum of the numbers in each row, column, and diagonal of the square.

Figure 4 shows some positive and negative numbers in a

square of the third order. The first number in the set is -3 and the common difference is 2. Hence, $S_{-3} = 2 \times 15 + (-3-2)3 = 30 - 15 = 15$ the sum of the numbers in each row, column, and diagonal of the square.

Figure 5 is a square of the third order filled with fractions.

$\frac{17}{8}$	$\frac{3}{8}$	$\frac{13}{8}$
$\frac{7}{8}$	$\frac{11}{8}$	$\frac{15}{8}$
$\frac{9}{8}$	$\frac{19}{8}$	$\frac{5}{8}$

FIG. 5

$2\sqrt{3}+7\sqrt{2}$	$2\sqrt{3}$	$2\sqrt{3}+5\sqrt{2}$
$2\sqrt{3}+2\sqrt{2}$	$2\sqrt{3}+4\sqrt{2}$	$2\sqrt{3}+6\sqrt{2}$
$2\sqrt{3}+3\sqrt{2}$	$2\sqrt{3}+8\sqrt{2}$	$2\sqrt{3}+\sqrt{2}$

FIG. 6

The first term is 3/8 and the common difference in the series is $\frac{1}{4}$ or 2/8. Hence, $S_{3/8} = \frac{1}{4} \times 15 + (3/8 - 2/8)3 = 15/4 + 3/8 = 33/8$, the sum of the numbers in each row, column, and diagonal of the square.

Figure 6 is a square of the third order filled with algebraic irrational numbers. The first term in the series is $2\sqrt{3}$ and the common difference is $\sqrt{2}$. Hence, $S_{2\sqrt{3}} = 15 \times \sqrt{2} + (2\sqrt{3} - \sqrt{2})3 = 15\sqrt{2} + 6\sqrt{3} - 3\sqrt{2} = 6\sqrt{3} + 12\sqrt{2}$, the sum of the numbers in each row, column, and diagonal of the square.

Figure 7 shows a square of the fifth order filled with integers

38	52	6	20	34
50	14	18	32	36
12	16	30	44	48
24	28	42	46	10
26	40	54	8	22

FIG. 7

in arithmetic progression. The first term in the series is 6 and the common difference is 2. Since the square is of the fifth order, $S = 5/2(25+1) = 65$ by formula (2). Hence $S_6 = 2 \times 65 + (6-2)5 = 130 + 20 = 150$, by the general formula (1).

In a square of the seventh order, $S = 7/2(49+1) = 175$.

In a square of the ninth order, $S = 9/2(81+1) = 369$ and so on.

Although magic squares have no practical application in mathematics it is believed that formula (1) makes it possible

to have much wholesome fun and needed drill in adding integers and fractions. It enables the pupil to know in advance the equal sums in the rows, columns, and diagonals of squares of the same or of different orders. The pupil can construct as many squares of the third, fifth, seventh, and so on to $(2m+1)$ orders as he desires to check for equal sums in the rows, columns, and diagonals. It is advised that practice be done on squares of the third order until De la Loubere's method of construction is well understood before attempting squares of higher orders. The formula (1) however is not limited to De la Loubere's method of construction only but is applicable to other methods if the reader becomes interested in learning them. De la Loubere's method of construction is given because of its simplicity. It must be remembered that the cells of these squares must be filled with numbers in arithmetic progression and that we may begin at any point in the series. This permits an indefinite number of squares of any order to be made. The length of the rows and columns, therefore, may be increased or decreased at will to fit the ability of the class enjoying the benefits of this activity in adding.

EDUCATION AT THE NEW YORK WORLD'S FAIR

In order that teachers, elementary and high school pupils, college students, and graduates working in specialized fields may find, among the thousands of exhibits at the World's Fair of 1940 in New York, those things which most closely touch their special interests, a Department of Public Education has been established at the Fair. This Department will offer many forms of service.

One important form of aid will be a series of leaflets. Two of these, "The Fair's Themes: A General Introduction" and "Science at the Fair" have already been issued. Four others, "Art at the Fair," "Exhibits for the Elementary School Child," "Social Studies at the Fair," and "Food, Decoration and New Products" will soon be available.

Teachers and school administrators may obtain copies of these leaflets by writing to the Assistant Director of Public Education at the Fair. Being intended for the use of educators, they cannot be sent in great quantities to pupils, though guides for student use will be issued later.

The Department of Public Education also maintains an Information Service. Teachers and pupils are invited to ask questions about exhibits, or send requests for itineraries suitable for various subjects and age groups. After the Fair opens, guide-material will be issued covering all special fields of study in which general interest has been shown.

Those planning to attend the Fair in large parties can obtain information on group-price reductions from the Fair's Special Admissions Department.

The Department of Public Education also invites suggestions concerning other forms of service which it may offer to help teachers and pupils derive lasting benefit from their visits to the Fair.

A DEMONSTRATION OF UNIFORMLY ACCELERATED MOTION

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Galileo's classic study of uniformly accelerated motion by means of a ball and grooved inclined plane has found its way into almost every high school Physics text. The usual method suggested for obtaining a set of data for distances traveled almost always leaves an unsatisfactory gap between fact and theory.

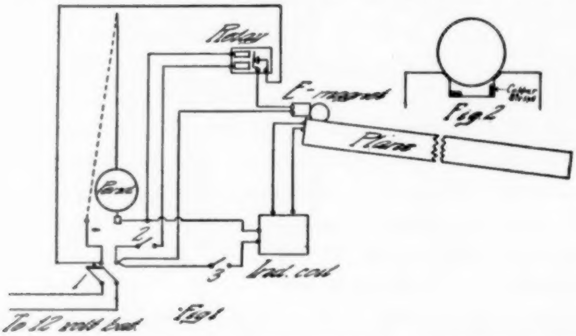
For the past several years an automatic and self-recording apparatus has been used with considerable success in the Physics department at Madison West High School. With the exception of two parts, the electromagnet and the grooved plane itself, the entire assembly consists of stock parts. The list of parts consists of a seconds pendulum with mercury contact, an induction coil giving a one-inch spark, a telegraph relay, an electromagnet, a twelve-volt storage battery, the inclined plane, a double pole switch, two single pole switches, a two-inch steel ball, and a number of the usual laboratory supports and clamps.

It is necessary to give a further description of some of the items used so that the entire setup may be clear. The inclined plane was made from a straight-grained piece of southern pine sixteen feet and four inches long and four by four inches in size. The groove was cut on a dado head power saw. The channel was about one inch in width and about three-eighths inch in depth. The edge of the channel was beveled with a hand plane. This beveled edge gives the bearing surface for the rolling ball and since it is sunk below the surface of the plane serves to protect the bearing surface from possible injury. In the bottom of the groove are nailed two strips of copper ribbon extending the entire length of the groove. The space between the ribbons is greater than the space between the ball and the ribbons when the ball is placed in the groove. Need for this is self-evident when the operation of the apparatus is understood. Figure *2* shows how the copper ribbons are placed in the groove. The copper ribbons were obtained by dismantling a choke coil from a mercury arc rectifier and removing the insulation. The size is one-sixteenth by three-sixteenths inches.

The electromagnet was made from one winding of a choke coil supplied by a radio battery eliminator. Its resistance is

about 270 ohms. Such high resistance is desirable. It draws about 45 milliamperes at 12 volts.

The telegraph relay has its insulating stud and contact points *interchanged* so that when the armature is drawn *over* by the relay electromagnet the "local" circuit is ***broken*** instead of closed as is the usual practice in telegraphy. The relay was further modified by supplying it with a light celluloid trigger supported from an upright post and resting on the armature. This trigger serves the important function of slipping down behind the armature when it was drawn over by the relay magnet and preventing the "local" circuit from recompleting after being broken. The necessity for this will also be understood when the operation is studied. In the complete assembly all parts requiring



special positions are held and supported by the usual clamps and rods found in all laboratories. The elevation of the raised end of the inclined plane is exactly eighteen inches above the other end.

Figure *1* shows all electrical circuits. In this figure all support rods and other items not needed to explain the operation are omitted. It will be observed that the single 12 volt battery serves for all circuits. On closing the double pole switch the electromagnet is energized (provided the celluloid trigger has been raised to release the relay armature to its normal position) and will hold the ball in the position shown. This circuit remains undisturbed as long as switch number 2 remains ***open***. The circuits through the relay electromagnet and through the primary of the induction coil are in parallel with each other and ***both*** circuits are in ***series*** with the pendulum through the mercury contact. As long as switches 2 and 3 are open the swinging of the pendulum has no effect on these circuits. The secondary of the induction coil is connected to the two copper ribbons lining the groove of the plane. On setting the pendulum into vibration and

closing switch number 3 a spark passes from one copper ribbon to the other by way of the steel ball at those instances when the pendulum sweeps through the mercury contact. On closing switch number 2 the pendulum will, on its next sweep through the mercury contact, complete the circuit through the relay electromagnet which by attracting the armature breaks the circuit through the electromagnet holding the ball and allows the latter to begin its journey down the plane. The release of the ball and the passage of the spark take place simultaneously. Because the instance of passage of the pendulum through the mercury contact is exceedingly short it is quite necessary to prevent the relay armature from recompleting the circuit and retarding the ball, since for this brief duration of time the distance traveled by the ball is not beyond the field of the electromagnet. The purpose of the celluloid trigger is now evident. The electric spark passes and marks the position of the ball each successive second for the duration of its journey down the plane.

The use of switches 2 and 3 will permit the testing and demonstration of each separate function of the apparatus. It will be observed that closing these switches does not release the ball or pass the spark but merely makes these circuits operative so that in the next passage of the pendulum through the mercury contact the desired functions occur.

To make a permanent record of the distances traveled a paper tape seven-eights of an inch wide and as long as the plane is placed in the groove of the plane. Care must be taken that the ball clears the tape at all points so that its speed is not retarded. This tape will be punctured by the sparks each second. In the writer's laboratory this tape was prepared from a strip of adding machine paper. The paper was pulled through a guide and cut by a safety razor blade. In obtaining a record for study the ball is rolled down the plane usually four or five times. The composite record is then treated statistically as shown by the appended table. Before the tape is removed the approximate positions of the punctures are marked. This is for convenience in locating them since the punctures are very small. As an aid in definitely locating the exact positions of the punctures a sheet of ground glass is illuminated by a light placed underneath. The pre-marked sections or the tape are examined on the sheet of ground glass by the help of a 4-inch magnifying lens. The limits of each group of sparks are marked and the total distances can then be measured.

A study of the appended table of data will show that the apparatus is successful in demonstrating its objective. This table is one chosen from numerous sets on file. In four or five trials over the same tape the punctures do not coincide exactly but are scattered over increasingly greater intervals. The average of the least and greatest distance is taken in each case. These distances are the only actual measurements made from the tape.

TABLE OF DATA

Time	1 second	2 seconds	3 seconds	4 seconds
Total distance traveled	27.2 27.6 54.8	109 110.2 219.2	246.9 248.2 495.1	439.15 442.50 881.65
Average	27.4	109.6	247.55	440.83
Distance traveled *each* second	27.4	82.2	137.95	193.28
Velocity at *end* of each second	54.8	110.07	165.62	
Gain in velocity per second (Acceleration)	54.8	55.27	55.55	
Distance ratios	1	4	9.03	16.04
Average acceleration is	55.2			
Substituting in the formula $v=at$ the theoretical velocities are:	55.2	110.4	165.6	
Substituting in the formula $s=\frac{1}{2}at^2$ the theoretical distances are:	27.6	110.4	248.4	441.6

In the treatment of the data the most difficult point for a class to grasp is how the velocity at the end of each second is obtained. It is characteristic of uniformly accelerated motion that the distance traveled during a time unit is numerically equal to the *velocity* at the *midpoint* of that time unit. The velocity at the end of the first second (or at the beginning of the second second) is the average of the distance traversed during the first second and the distance traversed during the second second. This method of treatment gives three velocities from this record. How the

acceleration is obtained is quite evident. The three accelerations are then averaged. Using the average acceleration the formulas $v = at$ and $s = \frac{1}{2}at^2$ are then applied to obtain theoretical velocities and distances. These are in excellent agreement with the figures obtained from the record.

THE BEST WORLD MAP—THE GLOBE

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There is only one true world map—the globe. The distortion which occurs in all wall maps arises from the difficulty encountered in representing a curved surface on a flat one. While a comparison of various projections readily shows how one varies from another, their truths and untruths are shown accurately by only one comparison—a comparison of the projection in question with the globe. If the projection and the globe are of the same scale the comparison is accomplished easily by a simple measuring device. When the two are made on different scales, the comparison is somewhat difficult unless one keeps in mind certain essential facts about the globe. These essentials are briefly stated in the following paragraphs, and then used in such comparison.

All north-south lines in the globe-grid are meridians and converge at the north pole and the south pole. The meridians are all the same length—180 degrees or about 12,500 miles. Every meridian is approximately one-half the length of the equator or one-half the length of any other great circle on the globe. The meridians are divided into equal sections by the equator and other parallels which intersect them. All angles at the intersections are angles of 90 degrees.

The east-west lines—the parallels—are all complete circles and so all contain 360 degrees but they differ in length. The equator is approximately 25,000 miles long. The length of the tenth parallel north or the tenth parallel south is about 63/64 of the length of the equator;[1] the length of the twentieth parallels, about 15/16 the length of the equator; the length of the thirtieth parallels, about $\frac{7}{8}$ the length of the equator; the length of the fortieth parallels, about $\frac{3}{4}$ the length of the equator; the length

[1] All figures are approximate and are given in this form for the convenience of quick and easy comparison.

of the fiftieth parallels, about $\frac{2}{3}$ the length of the equator; *the length of the sixtieth parallels, exactly $\frac{1}{2}$ the length of the equator;* the length of the seventieth parallels, $\frac{1}{3}$ the length of the equator; the length of the eightieth parallels, $\frac{1}{5}$ the length of the equator. Half the area of the earth lies between the thirtieth parallel north and the thirtieth parallel south.

Longitude is distance east or west of the prime meridian and is measured along the parallels. Each parallel, as has been noted contains 360 degrees. On the equator, which is a great circle, 360 degrees equal 25,000 miles. Therefore, each degree of longitude on the equator equals $1/360 \times 25000$ or approximately seventy miles. Since the tenth parallel north or south is 63/64 of the equator or about 24,600 miles, each degree of longitude there equals $1/360 \times 24{,}600$ or about sixty-eight miles. Figuring approximate lengths of degrees of longitude on the other parallels in like manner, a degree on the twentieth parallel equals sixty-five miles; on the thirtieth parallel, sixty miles; on the fortieth parallel, fifty-three miles; on the fiftieth parallel, forty-five miles; on the sixtieth parallel, thirty-five miles; on the seventieth parallel, twenty-four miles; on the eightieth parallel, twelve miles. At the poles—ninety degrees north or south—a degree of longitude is zero miles, for all the meridians converge at these points.

The 180 degrees of each meridian constitute one-half the length of the equator or 12,500 miles. Latitude is distance north or south of the equator and is measured along the meridians. Therefore, each degree of latitude is $1/180 \times 12{,}500$ or approximately seventy miles.

With this information in hand any novice can check on the accuracy of the main points in any map projection. As a specific example, measure the equator on a Mercator projection with a piece of string. Take one-half of the string measured to fit the equator and stretch it along the sixtieth parallel north or south. Is the sixtieth parallel on the Mercator one-half the length of its equator? No, the equator and the sixtieth parallel on the Mercator are of equal length. Therefore, each degree of longitude on the sixtieth parallels of the Mercator is twice as long as it should be. Next, check on the parallels. Are they the same distance apart? How does the distance between the equator and the tenth parallel compare with the distance between the fiftieth and sixtieth parallels? The same kinds of comparisons can be used as checks on all projections.

CONSERVATION EDUCATION IN RURAL AREAS

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The period of pioneering and exploitation of natural resources has ended. There are no fertile, untilled lands left; some of our resources are bordering decimation, and in general, our house is badly out of order. Awakened somewhat to these conditions we have realized that the natural resources have been utilized much as a young child uses his toys. But being older and hopefully wiser we think, we are endeavoring to do something before more wildlife follows the path of the passenger pigeon, or more of our agricultural areas simulate desert conditions.

The problem has been considered in various ways. Schools and colleges are offering conservation courses to emphasize the gravity of the situation. State and federal governments are attacking the problems on a number of fronts; the activities of the Biological Survey and Soil Conservation Service perhaps are most spectacular.

But the crux of the conservation program is the farmer. In his possession are the greater share of the remaining resources, human as well as material. Not only is the farmer producing our food wants and wildlife crops but he is likewise producing the human crop for the nation's new labor supply. The latter point is made evident in the recent population studies which have shown the following conditions, in brief:

1. Cities have always been dependent on migration from rural areas for the supply of labor in industry, but they are becoming increasingly more so.
2. Cities are not repopulating themselves. For example, in 1936 six cities, San Francisco, Oakland, Portland, Seattle, Albany and Utica had more deaths than births.
3. The poorest agricultural areas have the highest birth rate and also produce the greatest number of migrants.

We should be vitally concerned in this population trend which clearly indicates that we are depending more and more on the poorest agricultural areas, which are producing the most poorly educated individuals, for the maintenance of our future industries and democratic citizenry.

Regarding material resources, the farmer, whether he be a tenant, sharecropper or owner, uses the soil. This may be *wise*

use or *abuse*, the tendency having been almost generally toward the latter condition. Upon this same land much of our timber is grown and 85 per cent of all the hunting is done. Recent estimates report that 70 per cent of the country's wild fur crop is caught by farm boys.[1]

In view of these conditions what is being done to enlighten and assist the farmer in the betterment of his conditions?

Five agencies within the states might be considered as serving rural areas: the rural church, rural school, state agricultural college and extension service, farmers' clubs and cooperating federal bureas, and state departments of fish and game.

The Rural Church

The rural church probably contacts more of the rural population than any other single agency, and it undoubtedly wields a most powerful influence on rural family life. It is generally supposed that one's living ought to be harmonious with one's religion. But, literally speaking, eating oneself out of land and home, as many a farmer has done because of improper methods of soil management, does not make good sense, let alone sound religion.

We may have thought absurd the garden worship of some Chinese and Japanese, but these religions had some merit. To these people the soil was considered a precious heritage handed down from father to son, undiminished, and if possible increased, in productivity. A similar condition of soil preservation exists in some of the European countries, particularly Germany where farms have been maintained by one family for 400 years or more. These people take pride in their farms and the ownership is transferred to the son in the family who is best fitted to carry on the agricultural occupation.

Many of our rural people apparently lack a proper philosophy toward their farm lands. Maintenance of soil fertility or the soil itself, that he might pass on to his children a farm they would be proud to live on, is not a moral obligation with the farmer in general. In the opinion of O. E. Baker, chief of the Bureau of Statistics, and others, more could be done by rural churches to steer their people in proper thinking and building up a sound philosophy. Conservation is essentially a social attitude and unless an understanding desire is behind land improvement prac-

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[1] Hearings of Committee on Conser. of Wildlife Resources Seventy-sixth Congress H. Res. 65, 1939, pp. 45–46.

tices, their effects will react more as an anaesthetic rather than as a cure.

The Rural School

This agency, like the rural church, holds a key position in educating rural people in conservation activities, but little is being done.

Considering the country at large, rural school education is pathetic. While the urban schools have progressed with kindergartens, libraries, better teachers, and auditoriums, the rural schools have remained relatively static. Father and son have been taught in the same schools, with the same books, in many instances. The only change that has occurred is in class attendance and this often represents but a new generation of the same families.

Colleges for the most part are preparing teachers to teach in urban schools; consequently, the rural teacher is inadequately trained. But aside from all these difficulties the laboratory for the study of nature is there. Fields indicating erosion, rains washing away the school yard, and muskrats inhabiting the nearby marshes offer practical lessons in conservation studies. Unless the teacher is aware of these lessons however, they will not be taught. In view of this situation New York and California have issued leaflets on nature study to assist the pupil and teacher in understanding the outdoors. In sections of New Mexico and Arizona members of the educational staff of the Soil Conservation Service have ably assisted with instruction in soil and water conservation. In New York the Cornell Rural School Leaflet cooperating with the State Conservation Department has organized the Junior Conservationist league this year.

In general, a closer cooperation between rural schools and state conservation departments is desirable such as occurs in Delaware, Missouri, and Tennessee; between schools and extension departments as is exemplified in New Jersey. Many rural agencies have overlooked the potentialities of rural schools as sponsors of conservation education.

The State Agricultural College and Extension Service

Since its inception as a national program in 1914, the extension service has grown rapidly. In cooperation with the state agricultural college a program for youth and adults is provided in proper farm management practices. Demonstrations,

conferences, and publications are facilities used to further this program of rural education. In many instances federal bureaus have assisted with forestry and soil projects. Some states, notably Iowa, Minnesota and Ohio, have sponsored adult conservation short courses to discuss all phases of rural conservation projects.

Of equal importance is the youth program of 4-H clubs sponsored by this group. The training of youthful farmers is one of the most important functions of these agencies. In this respect there is a wide divergence in the kind of work that is being done in the different states. Some have seen the importance of educating the farm boy and girl in understanding nature about them; other states have considered farming very practically. Again attitude is all important. If the farm lad understands the effect of plant growth on the soil, the relationship between hawks and mice, and the basic needs of wildlife, he will derive an appreciation and enjoyment from nature with which his livelihood is so closely linked.

Ohio is one of the leaders in providing farm youth with a rich and varied nature program. Other states doing excellent work are Iowa, New Jersey, New York, Massachusetts, Michigan, Minnesota, Wisconsin, and West Virginia.

Conservation is proper land use; consequently, it ought to be the basis of every farm project. A specific program in conservation is not necessary inasmuch as effective activities can be worked out with the existing soil, home-making, and livestock projects. If a specific program is set up it should be balanced, providing activities on all phases of conservation that it may appeal to all types of individuals. In New Jersey officials attribute a large share of an increase in 4-H enrollment to the revamped club program which has provided a wide variety of nature and conservation activities.

The projects planned should not be too difficult to complete with a reasonable degree of success. In this category, might be mentioned some of the game propagation which is over-emphasized in a number of states. The difficulties of rearing birds challenge even the specialist, and unless excellent supervision is available it is better not to attempt such projects with youthful members. It is important that activities and projects outlined should not be mere suggestions, but rather, proven procedures applicable to the locality with explicit directions for carrying them out.

Needless to say, farm youth is more interested in projects which will provide cash returns. Planned activities such as muskrat management considered in Connecticut, sale of hunting privileges as in Ohio and Texas, and Christmas tree plantations advocated in Massachusetts are deserving of wider consideration.

The difficulties of most states in presenting an adequate conservation program are a lack of funds and leadership. The writer is inclined to believe that if we can obtain the leadership the necessary funds will not be difficult to obtain. Summer conservation camps offering instruction in all phases of nature have been successful in a number of states in furnishing needed leadership. Furthermore, every rural school teacher is a potential club leader, providing she can be assured of assistance and the fullest cooperation from the state leaders. New Jersey has proved this and last year the little state reported that 240 rural teachers were acting as 4-H club leaders.

Farmers' Clubs and Cooperating Federal Agencies

Among the various farmers' clubs the Farmer Cooperatives, Farm Bureaus, and the National Grange are perhaps the most conspicuous. These organizations are not sponsoring any particular programs in conservation unless they are associated with the Extension Service. On the whole these groups have served more as political and bargaining agents representing the sectional opinion on proposed land adjustment programs of the federal government.

The federal projects are sponsored chiefly by three agencies: the Soil Conservation Service, the Farm Security Administration, and the Agricultural Adjustment Administration.

The Soil Conservation Service has been applying land management programs on a demonstration basis to show the farmer what can be done to maintain land productivity so necessary for soil and wildlife conservation.

The Farm Security Administration has aided the farmer by supervised loans and grants. It has assisted him in improving his farm practices and/or in removing conditions of tenancy.

The Agricultural Adjustment Administration program is the largest of federal land agencies. Its purpose is to balance the supply and demand of farm products, to some extent, and at the same time remove marginal areas from cultivation. As is generally known payments are made for different types of con-

servation practices carried out by farmers. There has been a tendency to make adjustment payments according to the extent of proper land practices put into effect, and as a result some of the better farmers are receiving payments for carrying out practices, which they would very likely do anyway.

Whether all these farm programs are wise or not is debatable. At least they have made the farmer conscious that his methods of farming have not been what they should be and such an awareness is desirable.

State Fish and Game Departments

State fish and game departments are interested in the farmer because he owns the environment which maintains the wildlife that the sportsman wishes to shoot or catch.

Up to the last decade or two little attention was given the farmer, for game was plentiful and hunters comparatively few. Native species existed which were acclimatized to their locality; consequently winter feeding was hardly known. Natural predators kept deer, mice and other animals in check.

But more land was needed to furnish sufficient game for the increasing army of hunters. Although misunderstood, even today by many sportsmen, the farmer is becoming recognized as the principal custodian of wildlife. What many a hunter does not understand is that the emaciated rabbit or immature pheasant which was bought with his license fee funds requires food, shelter, and a certain ranging area for normal growth, and this environment is furnished by the farmer's land.

The pertinent question is whether the farmer is entitled to some compensation for the game obtained on his farm. A gentleman from Connecticut has well expressed an answer when he wrote that "a man can no more expect free game from a farm than he can expect free hay."

A number of state departments are assisting the landowner financially for the game removed from his farm. In Texas it is reported by Taylor that some farmers receive approximately fifty dollars for each deer removed from their land. Some of the progressive eastern states have been impressing upon the landowner the necessity of treating wildlife as another farm crop. Yet these same states are sending hunters on the farms to harvest the wildlife and expect to pay little or nothing for this desirable crop. In New York, the superintendent of game recently stated that the state could not possibly pay more than ten cents

an acre for the hunting rights on farmers' land. But if an acre of land can produce one pheasant, or one rabbit, or one quail it certainly ought to be worth more than ten cents; if it isn't, the farmer is paying too much for his haircut, his garage repairs and his clothing.

Most states have some cooperative plan of posting farm lands or refuges. Ohio deserves commendation in this regard for not only are the farmer-sportsman relationships amicable, as far as the writer has been able to ascertain, but the activities of rural groups are given a prominent section in the monthly conservation magazine. Michigan, New Jersey, Pennsylvania, Texas and Wisconsin might also be singled out for their efforts to enlighten the farmer on his position in the wildlife program.

The Missouri Conservation Commission is sponsoring a rapidly growing conservation club for youngsters. In an effort to assist the poorer children to participate in the activities the department distributed to rural schools bundles of old wildlife and outdoor magazines which had been contributed by city sportsmen.

In erosion control practices game departments have advocated that vegetation be planted which would also serve as food and shelter for wildlife. Some departments have furnished the farmer with shrubbery and seed grain.

Generally speaking, the farmer is willing to permit hunting on his land without fee, providing he is extended the courtesies considered proper. It must be recognized, however, that wildlife crops, if they are such, are incidental with most farmers who are trying to make a living from the soil. Viewed in this light, it ought to be reasonably clear that the farmer may not be too enthusiastic about increasing the rabbits and deer that eat up his vegetables, or the pheasants that feed on his grain. Only when the farmer can realize sufficient returns from wildlife will he make a conscious effort to improve the environment and increase the number of game animals.

Conclusion

It is evident that some effort is being made to enlighten the farmer on his position in the conservation program. The tendency has been, however, for each agency to push its own program. What is needed is a coordinated plan involving all agencies, probably of the type advocated by the Soil Conservation Service. Such a program of proper land use includes all

the problems of our natural resources, and the interdependence of these resources makes it imperative that we consider a conservation program on the broadest terms.

It has been written:

> . . . the farmer is the ultimate conservator of the resources of the earth. He is near the cradle of supplies, near the sources of streams, next the margin of the forests, on the hills and in the valleys and on the plains just where the resources lie. He is in contact with the original and raw materials. Any plan of conservation that overlooks this fact cannot meet the situation. The conservation movement must help the farmer to keep and save the race.[2]

So wrote Liberty Hyde Bailey in 1911, almost thirty years ago. And the conditions are even more true today.

[2] Bailey, L. H. The County Life Movement. 1911, p. 200.

DEMONSTRATING THE CYCLES OF AN ALTERNATING CURRENT

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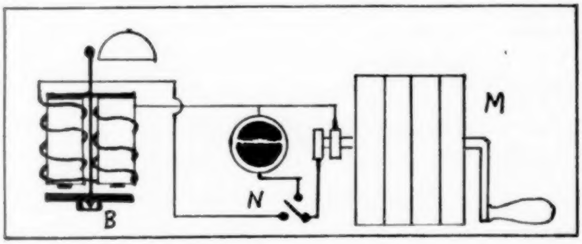
An assembly unit which always commands the attention of science classes, is one rigged up from a magneto *M*, a bell *B*, out of an old telephone set, and a neon lamp, *N*.

Since the magneto gives an alternating current, it will cause the plates of the neon lamp to flash alternately, first one semicircular disk glowing



Cycles of an Alternating Current

and then the other. It makes visible the notion of the cycle frequency, and the necessity for rapid alternations for satisfactory commercial use.

The bell is mounted with the pivoted clapper in view from the front and when the current is switched to it, a class may watch the change in vibration rate with the change in frequency of the alternations. Since this bell does not work on the interruption principle like the ordinary bell, it has a special interest.

[Editor's Note: How *does* this bell work? Is there any other practical piece of apparatus that operates like this bell? If you cannot answer, ask Editor Jones in Science Questions.]

INSTRUCTIONAL MATERIALS FOR A NEW COURSE IN PHYSICAL SCIENCE

H. W. Haggard
Wells High School, Chicago

When science at Wells High School was developed and organized as a core-curriculum field, a new approach to the problems in the classroom was necessary. The more advanced scientific facts and theories of the conventional general science course were deferred until the third year and developed under the heading of Physical Science. This allowed the new course in Introductory Science of the first year and the Biological Science of the second year to be built around a more socialized program of health, community needs, and pupil development and resulted in a more mature and comprehensive program for the third year. The course in Physical Science has been organized under general divisions as follows:—

1. An introduction to the scientific method used in physics and chemistry.
2. A thorough study of earth science, including
   a. Astronomy
   b. Geology
   c. Meteorology
   d. Conservation
3. A comprehensive study of simple and complex machines and their applications.
4. A complete examination of electricity and magnetism and the many applications used in modern life.
5. A study of transportation, communication, and the science of industry.
6. A survey of conveniences and advantages for successful living developed by science.

To motivate such a course for third year students requires considerable diversity of equipment, demonstrations, discussions, and reference materials. Particularly, reference materials must have a well defined set-up in order to function properly, and it is with this phase of the course that this paper is concerned chiefly.

An adequate room library for ready reference has proved the best answer to this problem. Just what constitutes the contents of this library has depended upon the local school, the environment, and the interests of the pupils. The library has had to be flexible and expansive from year to year in order to meet the varied types of activities which arose from building this course

about a program which is at once student centered and meets social needs.

A well organized and diversified library motivates the students to its use to a high degree of efficiency and results in a thoroughly comprehensive, well-developed, and scientifically stimulating course of study. A lack as to numerical quantity of materials does not necessarily constitute a barrier to developing a marked sense of appreciation of the achievements of men in the field of the physical sciences.

As would seem reasonable, a principal source of reference materials has been standardized texts in the major fields of the course, such as physics, chemistry, and general science. Just how many titles there should be in each subject division has depended upon the size of the group using the library, as well as upon the supply of such texts as are authorized by the Board of Education. No extra outlay or expense has been incurred in obtaining a sufficient supply of these books. Five to eight titles of each have been used, though possibly two or three titles would be satisfactory.

In regard to some of the more specialized divisions of science, it has been our experience that considerable care should be exercised in order to present a representative selection of reference materials of both a textbook and non-textbook nature. It is well to keep in mind that the book list of the average high school does not have a quantity of titles in geology, physical geography, meteorology, transportation, communication, conservation, and industrial science available for the exclusive use of a physical science course. Hence a very careful study of interests, aims and funds of the course needs was made so as to present the most effective choice of titles. Fortunately, loans from the school library for the duration of each unit have aided with respect to books of this nature.

In particular, some difficulty has been experienced in finding books whose language and viewpoint are sufficiently elementary and at the same time technical and scientific enough to stimulate interest in the subject presented.[1] In astronomy our aim is to have at least one authoritative text of an advanced nature available if any technical questions should arise. Then one or two scientific but simple references are chosen to present a clear idea of star groups, the solar system, astronomical phenomena, and astronomical instruments and observatories.

[1] See brief suggestive bibliography at close of article.

In various parts of earth science a few good descriptive books are utilized to supplement the up-to-date texts recently written. These books have been acquired from locally raised funds on an accumulative plan. A large quantity of these books has not been necessary to the success of the course. From various types of non-fiction books industrial, mechanical, electrical, communication and transportation subjects, an inexpensive representative supply has been obtained.

With many sources exhausted in search for the most effective list of materials, it was found that a number of phases of the course were still inadequately taken care of. Several additional sources were still possible. In the field of communication, American Telegraph and Telephone, Radio Corporation of America, National Broadcasting Company, General Electric and similar agencies had many free and inexpensive pamphlets, books, and articles concerning their respective connections with the field.

In the field of transportation, the main automobile manufacturers, steamship lines, air lines, and railroads have source materials which are invaluable to a physical science course. In the fields of conservation and industrial science, the governmental Departments of Interior, Labor and Commerce, issue a wealth of material from which to choose. Also, several information services, such as H. W. Wilson and Company, Vertical File Service, Classified Educational Guide, New York City keep a reference list of all articles and pamphlets on varied aspects of science. Finally, another source of information can be gathered from miscellaneous articles in various types of magazines and newspapers. The collecting of these pamphlets, articles, and the like may be used as the basis of a project for the more interested and keenly wide-awake students in the group.

A proper place in the physical science room for keeping this widely diversified accumulation of reference materials had to be provided. Some system for permitting these references to be used during class time as well as for overnight use should be worked out so as to have available the greatest number of them to the most students. The regular library card system is one effective method that might be used. If some dignified methodical system is not employed the effectiveness of the course may be destroyed.

The above course has been in effect for two years. The library has been partially completed and the results fairly satisfactory to date. Great interest, eagerness and enthusiasm has

been displayed in the use of such a library. Out of the room library, of 350 volumes, pamphlets, etc., as many as 110 items at one time have been signed up for overnight study, on a purely voluntary basis. No required homework seems necessary to stimulate this extensive use of the room library, yet highly varied and individualized written reports and class discussions have developed participation of all of the students. A strict understanding of the privileges and responsibilities of the proper conduct of such a library is well taken by the 300 students who use the classroom daily. There is a feeling on the part of both students and teachers that a carefully enriched reference library, as above, can and does make an inspiringly useful and broadening course worthy of the serious consideration of most third year students.

Recommended Books

Babcock, F. Lawrence, *Spanning the Atlantic*, Alfred Knopf.
Bartky, Walter, *Highlights of Astronomy*, University of Chicago Press.
Biddle, *Dynamic Chemistry*, Rand, McNally & Co.
Bisset, Captain J. C. P., *Ship Ahoy*, Private Publication, Cunard Line.
Davis and Sharpe, *Science*, Henry Holt & Co.
Dryer, Charles R., *High School Geography*, American Book Co.
Eckels, Shaver & Howard, *Our Physical World*, Chas. Sanborn & Co.
Fletcher, G. L., *Earth Science*, D. C. Heath & Co.
Henderson, *The New Physics in Everyday Life*, Lyons & Carnahan.
Hunter and Whitman, *Problems in General Science*, American Book Co.
Johnson and Lee, *Student Manual of Astronomy*, Rand, McNally & Co.
Pieper and Beauchamp, *Everyday Problems in Science*, Scott, Foresman Co.
White, W. B., *Seeing Stars*, Harter Publishing Co.
Wylie, C. C., *Our Starland*, Lyons & Carnahan.

SCIENCE IN THE PRIVATE SCHOOLS

The Annual Meeting of the Science Teachers' Section of the Private Schools Association of the Central States was held in Chicago last November. The discussions emphasized three points: (1) Testing and its relation to science teaching, (2) Books and non-text book material useful to science teaching, and (3) Classroom procedures and other real experiences in science. Other activities of the convention included trips to the Municipal Airport and to the Demonstration Physics Laboratory of the University of Chicago. The principal speakers were Mr. William Shultz, Cranbrook School, Mr. H. G. McMullen, and Mr. Glenn Blough of the Laboratory Schools of the University of Chicago, Lieutenant C. J. Von Yeast, and Mr. W. W. Strait. Mr. J. C. Mayfield presided. An excellent report of the meeting was prepared by the secretary, Miss Ida C. Wied of the North Shore Country Day School, Winnetka, Illinois. Lack of space prohibits publication of the report in full.

WHY TEACH MATHEMATICS?

Nelson A. Jackson
Mount Hermon School, Mount Hermon, Massachusetts

This question immediately divides into two parts, (1) Why do I teach mathematics? and (2) Why should mathematics be taught? How many of us have the courage to ask and honestly answer these questions?

1) Why are we, you and I, actually teaching mathematics this present school year of 1939–40?

Some teach mathematics for a certain number of dollars a week.

Some teach mathematics because there was an extra algebra class without a teacher, and it was assigned to the football coach who had a free period at that time.

Some teach mathematics because the subject was easy for them in school and college, and they thought they could teach it with a minimum amount of outside work.

Some teach mathematics because they love the subject, appreciate its beauty, and really find joy in showing boys and girls how to master the intricacies of the problem and theorem.

Some of us really teach, some of us muddle along, and some of us all unintentionally do real harm in the classroom.

2) Why should mathematics be taught?

We all know the answers given to this question by writers and theorists. Some of the reasons are—

a) Modern industrial mechanized life is founded on mathematics and could not exist and progress without the constant application of mathematical principles. It is, therefore, essential that young America should know its mathematics.

b) One finds great use for mathematical knowledge in the everyday affairs of life.

c) A thorough training in mathematics gives one an unusual regard for the truth.

d) Work in mathematics gives one habits of accuracy and logical thinking.

e) Mathematics is one of the oldest subjects in our modern curriculum; its cultural value is high; it is a language common to all people. It should be taught for its own beauty and interest content.

Let us examine somewhat carefully these reasons as to why mathematics should be taught. In this examination we shall

talk from the standpont of the traditional high school mathematics, i.e. algebra and plane geometry. We grant that every child should have some conception of numbers; should be able to perform the simple fundamental operations; and should have some knowledge of common and decimal fractions. All of this material is presented in the grades, most of it by the end of the sixth. Notice, I use the word presented, for in many cases that is all, many children never master these basic operations.

In the reasons listed for teaching mathematics there is much truth as well as much nonsense.

a) Modern industrial life could not exist and advance without the application of mathematical principles. This is true in so far as the technicians and engineers are concerned. But just how much does our mathematical knowledge help modern industrial life to progress? If we can make change, read the stock quotations and baseball scores, and figure out how much interest we actually pay on installment buying, we should be happy, for this amount of mathematical knowledge is mastered by the few and not the mass of our fellowmen.

Please note that no knowledge of algebra or geometry is needed by the ordinary citizen in order to do his part in our modern work-a-day world.

b) One finds much use for mathematical knowledge in the everyday affairs of life. This statement is not true when we speak of mathematics above the sixth-grade level. Just how many times outside your teaching have you had occasion to make use of your knowledge of the quadratic equation during the past year or for that matter during the past ten years? How many of us make any use of the theorem, "If two sides of a triangle are unequal, the angles opposite are unequal and the greater angle is opposite the greater side"? How long since you found it necessary to make use of the fact, well known to us, that a minus number multiplied by a minus number equals a positive number? It is nonsensical to say that we use our knowledge of high school mathematics in everyday life. We just simply don't, and that is the end of the matter unless we teach or have a technical job. And while we are at it, we might just as well include with algebra and geometry most of the courses as frequently taught in general mathematics, commercial arthmetic, and bookkeeping. Practically everything that has been said thus far will apply equally well to these so-called modern courses.

c) Mathematical training inculcates in one a high regard for truth. Does it? We all know too many well-educated crooks to be greatly influenced by this. Yet I have heard speakers extol the eternal verities of mathematical principles with the fervor of oldtime evangelists. Such enthusiastic preachers seem to feel that if teachers can teach Johnny the fact that the square of the sum of two numbers is always the square of the first plus twice the product of the two plus the square of the second no matter when or where the law is used, then Johnny will be filled with awe and wonder at the universal truth of mathematics and made to feel the necessity of living a truthful life. We know that this does not follow. As a matter of fact, Johnny's usual reaction is that it is just another blamed rule to remember which he will proceed to forget as soon as possible, and he will never give a second thought to the lesson on truth which was so carefully presented.

To teach a regard for truth should be one of the aims for mathematics teachers as well as for all other teachers. There seems to be nothing inherent in mathematics which makes it an unusual medium for training boys and girls to have a high regard for truth. It is the teacher and not the subject which accomplishes this.

d) Training in mathematics causes the pupil to form habits of neatness, accuracy, and logical thinking. All of us who have had extensive classroom experience know that this statement is not true, although it is frequently given as a reason for teaching mathematics. If teacher A requires her pupils in geometry to hand in neat, accurate, and well-arranged papers, this requirement is not in the pupils' minds associated with geometry but with teacher A. This care with papers will not carry over into the classes of teacher B unless it is also here insisted upon.

We hear much about accuracy in mathematics. If we are honest with our pupils, we must show them that outside counted numbers there is no such thing as absolute accuracy. All measured numbers are only approximate. We can find the exact number of apples in a bushel, but the weight of that same bushel can only be found to the nearest pound, ounce or fraction of an ounce; the exact or absolute weight cannot be found.

ΠR^2 is often regarded by emotional sponsors of mathematics as one of the eternal verities, but it is only an approximation. To be sure, the approximation is accurate enough for the engineering miracles of this modern world but still it is an approxi-

mation and not absolute. When a thinking pupil first realizes that π is the ratio between two measured numbers, that R is a measured number, that measured numbers are never exact, that, therefore, the formula πR^2 is not exact, that for the same reason all the familiar formulas for areas and volumes are not exact, then his mathematical thought processes receive a jolt and have to be revised. Right at this point the teacher has an unparalleled opportunity to open wide the portals and to point out to such a pupil the wonders of the modern mathematical world.

Logical thinking! Do we teach this in geometry? No, not often! There is some reason for considering geometry a good medium for training in logical thinking. However, my observation is that most of our pupils in high school geometry classes are too young really to benefit by the logic of the subject. For many of them it is a marathon in memory exercises. Here again it is largely the teacher and not the subject that furnishes the desired training.

The best exercise in logical thinking which it has been my privilege to observe for a long time was given by a teacher of a class in American History. He presented the dangers of propaganda to which we are all constantly exposed and taught his boys how to be on their guard, how to analyze and to draw safe and careful conclusions.

Neither the study nor the teaching of geometry makes logical thinking beings of any of us. Outside the classroom mathematics teachers are notoriously illogical. Why say then that the study of mathematics makes one logical?

e) Teach mathematics for its own beauty and interesting content. Don't drag in the other reasons to bewilder the layman. All the reasons mentioned will be by-products of good teaching whether of mathematics or any other subject.

Mathematics is one of the oldest subjects in the high school curriculum. It has developed slowly through the centuries from the earliest prehistoric conception of numbers to the present vast basic science which does underlie our modern world. With it are connected such names as Euclid, Pythagoras, Thales, Newton, Napier, Steinmetz, and Einstein.

Let us, therefore, teach algebra, geometry, and trigonometry for their own sake. If we know our subject, are enthusiastic teachers, feel that we have something worthwhile to give to the younger generation, then we shall kindle in our students a real

love for mathematics. They will come to recognize the beauty and symmetry of form; they will learn the enduring satisfaction of accomplishing a hard task for the sake of itself; they will learn how the history of the development of mathematics has gone hand in hand with the history of the development of mankind; they will see why a knowledge of mathematics is necessary if one is to understand the intricacies of our modern world; in some instances they will be influenced to go to college and technical school, and as the result of such teaching they will live a fuller and more satisfying life.

I easily become enthusiastic over the teaching of mathematics for its own inherent worth. However, our classes in algebra and geometry should be composed only of those boys and girls who have an aptitude for mathematics and a real desire to take these courses. From the scores made on some of the modern aptitude tests and the cumulative record of progress made in the grades, one can judge with a fair degree of accuracy any given pupil's aptitude for mathematics. If a pupil has the aptitude, then he should be given the benefit of the training.

Just now when a goodly number of states no longer require that algebra and geometry be taught during the high school course, when many of our leading educators advocate the elimination of the traditional secondary school mathematics, when some colleges no longer require algebra and geometry for entrance, we must not go to the extreme of denying the knowledge of these subjects to those who can pursue them with profit. Just because Tony is the son of the corner fruit dealer, and there seems to be no likelihood of his going to college, he should not be kept from these courses, if he has the aptitude for them. He may become a leading engineer or a scientist of note, if given the opportunity. On the other hand, John, from a home of wealth and influence, should not be forced into these courses if he lacks the aptitude for them.

We are teachers of mathematics. What kind of a job are we doing? Do we teach with vigor and enthusiasm? Do we regard each day's work as a challenge? Do we inspire our pupils with a desire to do good honest work? Do our boys and girls in their serious moments (and they really do have such times) consider our classrooms as places where something worthwhile is being done? If these things are true of us, then we are teachers and not time servers, and our results will be satisfactory. But we can't get results unless we give ourselves heart and soul to our teach-

ing, and consider that the one important thing for us to do. The method used is not essential. We shall accomplish results if our heart is in our work; otherwise, we shall be but mediocre teachers or failures.

The moment we lose interest it all becomes monotonous, and our teaching is but a treadmill existence. We have no right to continue in the classroom, if we have no interest in the work.

We are dealing with human souls not merchandise. We are influencing the lives of boys and girls. Our responsibility is heavy. Through the medium of mathematics we have the unparalleled opportunity of helping boys and girls to develop into men and women who in a few years will be capable and willing to take on the job of running this old world of ours.

PRINCIPAL VALUES OF INVERSE TRIGONOMETRIC FUNCTIONS

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The discovery that students who have studied from different texts have contradictory definitions for the principal values of the inverse cotangent, secant, and cosecant suggested an analysis of the definitions given in a representative sample of elementary texts. Twenty-five recent trigonometry texts were examined. In two texts inverse functions were not mentioned; two mention inverse functions without discussion of the principal values; one text uses the term principal value without definition.

By far the most common definition is essentially, "The numerically smallest angle defined by an inverse trigonometric function is called the principal value of the function. When two equally small angles are of opposite sign, we select the positive angle." This definition was found in eleven books, the same definition with the omission of the second statement, in four books. Although the second sentence in the definition is certainly needed to remove ambiguity, we may charitably assume that the intent of all fifteen books was the same. This definition makes the principal value of the inverse cosine and inverse secant fall in the interval from 0 to π, while the principal values of the inverse sine, tangent, cotangent, and cosecant fall in the interval from $-\pi/2$ to $+\pi/2$.

Five texts, including at least one that is rather widely used employ definitions which for the inverse sine, cosine, and tangent agree with the general definition just given; but which differ for the inverse cotangent (the principal value is defined as falling in the interval 0 to π); the inverse secant and the inverse cosecant (by definition the principal value of arc sec x or arc csc x lies in the interval 0 to $\pi/2$ or in the interval $-\pi$ to $-\pi/2$ depending upon whether x is positive or negative). These definitions, which one author states seem apparently artificial, are sometimes explained as justified by the needs of the calculus.

Since justification for the second set of definitions seems to be their use in the calculus, a representative sample of introductory texts in calculus was examined. Of twenty such books, five do not discuss the question of the principal value of these functions; four avoid controversy by not making use of the inverse cotangent, secant, and cosecant; five use the first definition quoted above (smallest numerical value, with provision in case of ambiguity); five use the second set of definitions given previously; and one, to be different, agrees with this set in placing the principal value of the inverse cotangent in the interval 0 to π, but places the principal values of arc sec x and arc csc x in the interval π to $3\pi/2$ when x is negative.

It is not the purpose of this note to argue for or against any set of definitions; it may be noted that although the matter may seem trivial in a beginning course, in later work the proper interval may be important. If nothing else, there is confusion when students in the same course disagree, with apparent authority for their disagreement.

A very interesting point is that in only one text was there a statement to the effect that there might be any difference in the selection or definition of the principal values. Authors might at least be kind enough to add a footnote and save the student trouble later on. If mathematics is supposed to be an exact science, how can we justify a definition contradicting a definition of the same term in another reference, with no statement regarding possible alternatives?

Education is the talisman that brings you luck when all else fails. Without education your money is worth nothing and life's chief prizes are out of your reach. Education is the difference between useful and comfortable living and the sort of living that merely checks off the days as they pass, without pride of accomplishment or hope for the future.

THE BIOLOGY CLUB AS A CHAPTER OF THE JUNIOR ACADEMY

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An article in the *Reader's Digest* a few months ago brought realization of the increasing interest in extra-curricular science activities, particularly in the so-called Junior Academies of Science. The Academies of St. Louis and Missouri are merely representative of a movement which has spread over one-third of the nation. This paper will consist of two divisions—first a description of the organization and functioning of the St. Louis and Missouri Junior Academies; and second, a critical analysis of the justification for such groups, especially with reference to the biological sciences.

As long as ten years ago, Mr. W. F. Shay, a gifted biologist and teacher at Normandy High School, tried to interest the Academy of Science of St. Louis in starting a Junior Academy. His idea took hold very slowly. Committees were formed, but no definite action was taken until early in 1937. In that year the state academy was to hold its convention in St. Louis, and they desired an exhibit of junior work as an example to the rest of the state.

With this incentive the Junior Academy was finally organized in St. Louis under the leadership of W. D. Shipton, Geology professor at Washington University, and secretary of the senior Academy. There were seven charter chapters, representing one city public school, Soldan; two county public schools, Normandy and Webster Groves; and one private school, John Burroughs. Four of these chapters were previously organized biology clubs. The first exhibit aroused much interest and led to the formation of four new chapters the next year.

Mr. Shay lived to see the success of the 1938 annual meeting in St. Louis and to manage the first convention of the Junior Academy of Missouri at Rolla. Over two hundred St. Louis high school youngsters travelled the hundred miles and had a profitable time. Less than a month after this, Mr. Shay fell dead of a heart attack while on a botanizing trip and left his work to be carried on by others. This is being done successfully, as there are now over twenty chapters with a total membership of nearly seven hundred.

A brief summary of rules and customs might be of interest here. Any person under the age of 21 is eligible for membership in the Junior Academy of St. Louis upon payment of the dues of fifty cents a year. He may join individually, but so far all the members are united in various chapters, each under the leadership of a sponsor who is a member of the senior Academy. A chapter must be named after a great scientist.

Each spring there is an annual meeting with papers and exhibits by the members. No prizes are awarded. The intention is to encourage work for the love of it and to offer opportunity to share ideas with other interested individuals. It is attempted to reduce the spirit of competition to a minimum. The A.A.A.S., however, has recently awarded two honorary memberships each year and these are given by the sponsors to the boy and girl who have shown the most promise.

Several general meetings are held during the year, some presenting outside speakers and others discussions among the members. These are all presided over by a president who is elected by a cabinet consisting of two representatives from each chapter, except when there is more than one chapter in a school, in which case only one representative is sent.

A paper, the *Junior Academy Journal*, is published about four times a year. This year a "Project Kit" has been assembled which includes descriptions of the projects of the members in all fields. It is offered for sale to outsiders for twenty-five cents a copy. Other Academy activities are radio broadcasts and trips to places of interest.

The chapters of the St. Louis group become members of the Missouri Academy upon payment of $1.50 per chapter. So far, a satisfactory financial set-up for the state organization has not been attained. Miss Long of Normandy, who assumed the chairmanship of the Missouri Junior Academy after Mr. Shay's death, has given generously not only time but money in order to organize the state chapters on a sound financial basis. Much has been achieved but the final success is still problematical. The greater St. Louis groups, however, pay for themselves and seem firmly established.

What are the benefits to be derived from such organizations? These can be considered from three viewpoints—the student's, the school's and the community's.

First of all, a member of the Junior Academy is provided with an incentive to plan and complete a piece of work. In doing so he has the use of the school facilities and the benefit of the sponsor's

time and experience. This is a step toward doing something for the superior student who is all too often neglected in our vain attempts to educate the uneducable. Two of the boys at Soldan wished to try out soilless plant culture. They could not have afforded to buy all the materials themselves, but with the use of the school's tanks, distilled water, and chemicals, were able to carry on a successful experiment.

Many other instances could be cited. The use of the microscopes after school hours costs the school board nothing, but is an invaluable opportunity for the child. Of course, this sort of thing can go on without a Junior Academy if the teacher is willing; but my experience has been that five or six times as many students take advantage of these opportunities since they have the stimulus of the Junior Academy.

Prominent scientists are frequently willing to speak before young people. Speaking before many small groups would take too much of their time, but an organization such as the Academy can guarantee large and appreciative audiences. Our group in St. Louis has had the advantage of hearing eight or ten of the best scientists in the city and even some nationally known men, such as J. W. Lippincott, the explorer, who spoke before the group last year. In addition to the men making these addresses, many of the members of the senior Academy give individual help to students interested in their particular fields.

From association with these men, and from working on their own projects the students get a deeper appreciation of the work of scientists. Measuring and weighing materials carefully, trying to find the reasons for failures, and seeing how far short of the best their own results are, tend to give them more respect for craftsmanship—a quality sadly lacking in today's world.

Acquaintance with the progress of science is gained from a study of various publications for help in their projects and in doing library research for articles for their Journal. Even those who do not take an active part get something from merely reading the Journal. It has been the policy to publish in each issue a biography of one of the scientists whose names the chapters bear. This also fosters an appreciative attitude.

The radio stations have been generous in allotting time for Junior Academy broadcasts. This field has been neglected for the last year, but before that several successful broadcasts were presented, among them a play from SCHOOL SCIENCE AND MATHEMATICS.

Being a large group also gives the members the advantage of

cheaper transportation rates. As a result of this, nearly three hundred children saw Onondaga Cave, 75 miles from St. Louis; two hundred went to the convention at Rolla, 100 miles away; and nearly a hundred took the overnight trip to Springfield. This sort of activity is the best kind of training in geography, especially in problems concerned with industrial development and conservation.

No matter how brilliant an individual may be, he is not leading a complete life without pleasant relations with others. The Junior Academy helps forward such relations by giving the members a chance to exchange ideas with congenial persons and thus encourages a truly cosmopolitan attitude. This sort of contact is invaluable in fostering the real democratic spirit. The rich students from the expensive private schools learn to respect the ability of the people from the poorer districts who have had a hard time scraping together even the moderate dues of fifty cents. Those from the poorer districts, on the other hand, get to see some of the most expensive modern equipment. The antagonism of the "country jake" to the "city slicker" and the reciprocal contempt, is considerably lessened by more intimate acquaintance. Charles Lamb spoke wisely to a friend who asked him why he hated a certain person when he hardly knew him. Lamb answered that if he really knew him he couldn't hate him.

The school gains in at least two ways by having a Junior Academy chapter. Much of the members' work can be used as class demonstration material. One student mounted a whole set of chick embryos, another a series of dissections of vertebrate brains. Some are preparing microscopic slides while others are making clay models. This does not take business from the supply houses, but serves as additional equipment which the schools could not afford to buy. The bringing in of prominent speakers is also an advantage to the school as a whole, since guests are always welcome.

The chief benefit to the community comes from the encouragement of the broad cosmopolitan attitude. Science knows no national boundaries. The more people develop this spirit, the more chance there is of avoiding a conflict such as is now raging in Europe.

St. Louis has a special aim which it hopes the Junior Academy will help to accomplish. For years we have been working toward the establishment of a Natural History Museum. Valuable archaeological finds uncovered within fifty miles of the city

have had to be sent to New York or Washington because we have no place to house them. It is hoped that interesting the youth will aid in the attainment of this end.

Although the Junior Academy is such a worthwhile undertaking, there are still at least two knotty problems of management—that of finances, and that of sponsor time and energy. Carting exhibits from place to place costs money, more than the group can afford. Until now the sponsors have used their own cars or hired trucks. Miss Long brought some of our exhibits to this meeting in a rented trailer. It is hoped that this problem will be solved by a Junior Academy endowment fund, the interest from which will be sufficient to pay such incidental expenses.

Sponsor time and energy is a real problem. At present the sponsors not only supervise their own chapter activities but attend to the organization work and getting out the Journal besides. While the children do all the actual writing and editing, still the sponsor has to proofread and see that everything is done. Radio broadcasts are also an extra burden. This situation could be remedied if a few adults who are not chapter sponsors could become interested enough to act as coordinators—arrange for the general meetings, supervise the Journal, and take charge of radio broadcasts.

With these problems solved, the Junior Academy should go far in its aims of offering opportunity to students for creative work and congenial companionship. In so doing, it will become an important factor in the development of science and a peaceful world.

JUNIOR COLLEGE NEWS NOTES

Enrollment in junior colleges in the United States has doubled in the past seven years. The increase in the past year was from 155,588 to 196,510 or 26.4 per cent.

The largest junior college is the San Bernardino Valley Junior College in California, which has 8,317 students. This number includes 7,499 special students, most of whom are adults. An extensive adult education program is offered by eight California junior colleges. Los Angeles City College, with 6,687 full-time students, has the largest full-time enrollment of all junior colleges in the country.

There are 33 junior colleges in the country with enrollments of more than 1,000. The size which is most general is between 100 and 200, in which group there are 153 reported. There are 212 junior colleges with enrollments between 200 and 1,000. The average for all is 349.

PITFALLS AND PEAKS OF NATURE TRAILS

GLADYS LODGE
Briarcliff Manor, New York

"A nature trail!"

"Could we really make one?"

"Can we start right away?"

Such were a few of the comments from the children of Briarcliff Manor when the idea of a nature trail was suggested. The enthusiasm was great. Property owners as well as parents, teachers, and principal were cooperative. We laid the trail through a field, into a sunny swamp, along a brook, by a stone quarry, and into a wood. It was decided that each student should choose, along the trail, one thing of scientific interest to study, and should label this with a sign giving its name and one scientific fact about it.

In general there was wide variety in the topics chosen and the signs made to mark them, though the labels were all planned to blend with the scenery, and green arrows were used to mark the trail. Sometimes the signs were illustrated by the children's own drawings or by clippings. One boy did a good piece of work with goldenrod galls. He cut one in half, exposing the worm inside, and varnished this to hold the worm in place. He then varnished an uncut gall which showed the exit hole of the worm. These he wired to either side of his sign and put his explanation in the middle.

Considering the freedom of the children to roam about, the application to business was good. As our work on the trail advanced, there was improvement in outdoor manners, in the enjoyment of natural things in their natural environment, and in the use of the library, but in the field of science the gain was too small. There were unforeseen pitfalls that made the value of the project low.

I was disappointed to find that the children showed much more interest in making signs than in the scientific aspect of the subject. Some youngsters simply copied on their signs the first thing any book said about their specimens, without having an idea of what it was all about, some stood around not knowing what to do, and others dashed ahead making all kinds of mistakes. Even the little information called for did not meet my expectations.

When I think how I tried to be everywhere on a half mile

of trail, besides helping children with references, and keeping paint off their clothes,—when I think of how those children came and went, in and out, pawing over pamphlets and clippings, going to the library for further information and stories, digging to find the bottom of a dandelion root, diving in the water after a frog, or sinking in the mud to place a sign in the middle of some iris,—I wonder that the school survived it.

As for the attempt to leave a permanent trail, to the youngsters that was the most discouraging part of all. One morning when we were no longer working on the trail, we found our beautiful entrance arch knocked down and broken to pieces. Many of the children found their signs torn down and defaced. They wanted to bring in what were left so they would not, also, be destroyed. I told them they might.

My own disappointment in the venture set me to analyzing my mistakes. This analysis proved so useful to me that I tell about it and its subsequent developments in the hope that it may help others avoid the pitfalls and carry on to the peaks of outdoor teaching.

I realized that I, myself, had had the wrong aim. I had conceived the idea of having a trail with signs on it, the making of which would teach scientific principles. This, in itself, gave the wrong center of focus. The scientific principles had been only secondary to the making of the trail. I had hoped to organize the information gathered by the children into a few principles and concepts of science, but I had allowed such a wide range of subject matter that it was impossible to do it. Besides, this wide range of subject matter, together with the vast territory covered, was more than one teacher could keep track of properly in a group of thirty-five children. The facts dealt with were too elementary and the method of obtaining them too difficult, considering the little help I could give each one and their lack of previous experience in the method used. In the making of signs, which was their greatest interest, I had not the time nor was I properly trained to teach good techniques. Much time was wasted.

With this analysis and the firm conviction that child health requires fresh air and sunshine, I laid plans for my next outdoor teaching. Keeping scientific principles in their rightful place of first importance, I surveyed our environment for material that could be used to illustrate and teach them.

The results were gratifying. Seed formation, distribution, and

growth were illustrated in their entirety by the trees, especially the red maples, beginning in the spring when these first signs of growth are such a thrill. Then there were the ferns, horsetails, and algae, all without seeds, introducing different methods of propagation as well as technical classification. It was only a step from a dry field with fine-leaved grasses and weeds to a moist area with lush skunk cabbage, ferns, and Jack-in-the-Pulpit. The dandelions on the lawns had deep roots, the saxifrage on the cliffs had shallow ones. Here was a lichen-covered rock with moss at its edges and ferns beyond that, also a rotting log crumbling into earth, residual soil in the making and easy to understand, while a cut in the bank exposed transported soil. Here was a rotting log full of life. Yes, and it also exposed a tight and loose knot, so that we could slip the surrounding wood off the old dead limb. There were trees in the ravine with long branchless trunks that reached up till their tops were as high as the tops of those on the bank, while those in the field were short and branched all the way to the bottom. There were water striders that skated about never breaking the surface for air, fish that stayed under, and algae that floated to the surface, filled with bubbles challenging investigation.

These were just a few of the illustrations of important scientific principles found everywhere. The task was to organize the material and use it intelligently. With this in mind, I started to teach science instead of trying to make a "belabled" trail.

A trail, however, was soon worn along the creek bed where rocks were placed to improve the walking, and, wonder of wonders, disgust and contempt were developed for the way in which humanity had filled the ravine with debris. Bushels of refuse were removed by the children, and even when we found fresh papers and refuse, due to the fact that the trail had become a popular lunching area, the majority of the children resented this and cleaned them away.

That year I had the periods before and after the noon hour to devote to the first six grades, with all of whom I planned a course of outdoor work. There was hardly a fine day that I did not eat lunch on the trail with one of these grades. We considered what foods were best to have in our lunches, thus learning about vitamines, minerals, tissue repair, and energy. We became one of the animals using oxygen to oxidize our food. We learned how and where to build and how to care for small fires, what refuse could be burned and what could not. We gained a certain

amount of responsibility for the other fellow's welfare, learning not to climb above the group and knock rocks down on those below, and also learning to keep track of those behind us when traveling. Sometimes we tagged plants to keep records of their changes from week to week. A few permanent signs were made outside of class, but no school time was given over to sign making.

In many instances the grade teachers used the science interests as subject-matter for writing as well as reading. This was a very important supplement to the work, as it helped the children use and organize the knowledge they had gained, and also showed me how well they had or had not mastered it.

It was pleasant teaching and easy. After careful planning of the first lesson to insure interest and activity for every one, the work progressed on the strength of its own appeal. The outdoor laboratory was ready and waiting, and no lesson ever ended without enough things hanging fire to keep us busy for days.

Many children went off on tangents of their own, but there was always some central theme, such as soil formation, adaptation, or the like, on which the whole class worked and which held them together. In planning each trip I provided for the following:—observation and explanation of specific natural phenomena under my guidance, tasks and observations for children to perform by themselves, discussions, time to eat, and some free time besides. No one ever knew or seemed to care when the noon hour began or when it ended.

CONFERENCE ON CONSUMER EDUCATION

A regional conference on consumer education will be held on the campus of George Peabody College for Teachers on Friday and Saturday, May 17 and 18, 1940. The major themes include a consideration of basic questions in consumer education and current practice in consumer education. There will be two general sessions which will include addresses by John Cassels, Stephens College, Edna J. Orr, Alabama Polytechnic Institute, J. J. Oppenheimer, University of Louisville, James E. Mendenhall, Stephens College, Gordon McCloskey, Alabama College, and Leland Gordon, Denison University. The dinner meeting will be addressed by Donald Montgomery, Consumers' Counsel Division, Agricultural Adjustment Administration, Washington, D. C. The Friday afternoon sessions will consist of seven discussion groups.

Since this is the first conference of its kind in the southeastern region, a representative gathering of teachers and others interested in consumer education is expected. The conference is being conducted in cooperation with the Institute for Consumer Education at Stephens College.

FUNCTIONAL THINKING*

E. R. Hedrick

Vice-President and Provost, University of California at Los Angeles, California

Many have spoken on the theme of *functional* emphasis in Mathematics. I myself have done so so frequently that I may repeat myself today. In its explicit formulation, this principle in Mathematical teaching goes back to Felix Klein, whose potent voice advocated it at about the turn of the century. In this country major emphasis was laid upon it by the Committee on the Reorganization of Secondary School Mathematics, appointed by the M.A.A., under my own presidency, in 1916. Its report, published with the aid of the General Education Board about 1920, is known to most of you as the most constructive effort ever made toward the improvement of mathematical teaching in this country. It emphasized the fact that functional thinking is the one unifying element which pervades all proper teaching of mathematics.

Yet, twenty years after that report was issued, the greater part of mathematical teaching avoids functional ideas. Many of the most popular textbooks show little or no appreciation of it, although most of them state *in the preface* that they do so.

What is wanted is not that lip-service. It is not in the *preface* that we seek evidence, but in the body of the text, on every page, in every list of exercises and problems, in the attitude toward formulas and toward emphasis on functional situations. We do not desire to see the *word* function at all. What we do want is to see more recognition of the *idea* in every topic in every subject in the mathematical curriculum.

It is my purpose today to illustrate how this may and should be done, and to give concrete examples of elementary and advanced kinds, in a variety of subjects.

First of all, let me urge that there are two different ways in which functional thinking is done, both in mathematics and in the world. From an elementary standpoint, the idea of function is that one thing (usually a quantity) *depends upon* some other thing (usually a quantity). I know very well, of course, that a refinement of this description is required for advanced work, but

* Presented before the Mathematics Section of the Central Association of Science and Mathematics Teachers, Chicago, Nov. 24, 1939.

I am speaking now wholly of elementary ideas and elementary courses.

Whenever we discuss the dependence of one quantity upon one or more others, we are talking about functions. What is desired, then, in all courses in mathematics, is that great stress should be laid upon the dependence of quantities on others, and upon the way in which the one depends upon the other.

In such discussions of *dependence*, we do not necessarily know, in every case, the precise formula which connects the one with the other quantity. Indeed, the first question that arises, both in mathematics and in life, is the question "*On what other quantity does this* (the desired one) *depend?* If we can settle that question, the second question—which is quite a different one—arises: "*How does the one depend upon the other?*"

In the world, in problems of life, in problems of science, the one question is quite as important as the other. We may sometimes be able to answer the *first* question, which I shall call the *qualitative* aspect of functionality, but not the second one, which I shall call the *quantitative* aspect. Thus the average weight of a child depends upon its age; curiously enough it decreases for a few days after birth, and then in general increases with age, but the relation is not a simple one and I do not believe that any simple formula is known. At most we can give statistical information and we can draw a rough graph. A similar situation exists, for example, in the case of air-resistance to a body in motion, say an airplane. It is generally known that this resistance increases with the speed. We do know some statistical facts, but no formula exists, and only a rough graph can be drawn.

Such situations are very common in the world outside of the classroom, and in physics. Even in instances in which formulas are known, as in the case of the amount of a loan of money at compound interest. Thus, the very mention of this causes you yourselves *first of all* to think of the very quick growth at compound interest, and perhaps of a crude graph which rises very rapidly; only secondarily do you—even you—think of the precise formula, though you know it quite well. For the majority of people, the formula might be difficult, but the quick rise that occurs at compound interest is sensed by all who have even mild ideas of business. Such thinking, even when it is not precise, is to be classed as functional thinking, at least in the qualitative aspect.

That people do not think clearly about quantities without

some training even in the qualitative sense is very evident. Notable examples exist in which even very intelligent men failed to see that a quantity of great importance to them depended on some other quantity. Thus for centuries the Roman engineers built aqueducts to bring water for domestic consumption and for irrigation, and they measured it solely by the size of the opening through which it flowed. Apparently the first man to observe that the amount of water depended also on the *pressure* (the so-called "head") was Leonardo da Vinci. He demonstrated it and gave a formula that is still used in hydraulics. Curiously enough, centuries after his time, the same error was made by Americans in our own West, when irrigation began: water and rights to water were bought and sold for many years in the nineteenth century, in California and elsewhere, on the basis of a unit known as the "inch," which meant the amount of water that would flow through an opening of one square inch, without taking into account the pressure. Many bitter lawsuits are still in the courts of the West on account of this error.

If reasonably bright men, capable of constructing such serious engineering works as aqueducts, could fail to notice that a quantity in which they were vitally interested (the amount of water that flows through an opening) depends upon another quantity (the pressure), it is evident indeed that people will not always judge correctly that one quantity depends upon another, even if they are vitally interested. To quote a more homely example, the amount of matter in a given hen's egg depends upon its length (or other dimension), but millions of women follow cooking recipes that call for a given number of eggs without regard to their dimensions. Perhaps they think—perhaps *you* think—that the differences between eggs is at most small. You might calculate the difference if the linear dimensions differ by one-fifth, which is not uncommon.

Sir Oliver Lodge in a delightful but unfortunately little-known book entitled *Easy Mathematics—Principally Arithmetic*, discusses the popular inability to think out the relations between quantities. Some of his observations are quite humorous. Thus he inquires "If a horse can bear a load of four hundred pounds when standing on four legs, how much can he bear when standing on one leg?" He is here emphasizing and ridiculing the thoughtless use of proportion.

The modern theory of education emphasizes, and justly, the integration of related thinking. For mathematics, this means to

relate the work with such sciences as physics and engineering and with the activities of ordinary living that have to do with quantities and the relations between quantities. I have tried to make clear that there is a great deal of ordinary living which does have to do with quantities, and with those relations between quantities that constitute functional thinking. Such instances as those I have mentioned, and such other widespread activities as installment buying, purchases on mortgage, financial dealings at compound interest, life insurance, pensions, and many other life-activities depend vitally on such functional thinking. To deny this, to say that problems of life have no vital connection with relations between quantities, is to display either total ignorance of life and its affairs, or to stand convicted of a stubborn insincerity that can be based only on prejudice against mathematics and functional thinking.

It is not always easy, however, even for those of us who are favorable toward mathematics, and who know technical algebra, to see the connections that exist with everyday affairs. Thus, as I pointed out in another discussion recently, a report of the Education Policies Commission that is on the whole quite favorable toward mathematical teaching mentions the formula for $(a+b)^2$, and, by implication, that for $(a+b)^3$, as among those of doubtful connection with affairs of life. However, as I have insisted elsewhere, these very formulas are directly involved in such questions as the error in areas and in volumes due to errors in measurement, the retention or discarding of figures in computations, and the estimation of differences in volumes (or areas) due to differences in size, as in sizes of oranges. This instance is illustrative of the difficulty which sincere and informed persons experience in *transferring* their knowledge to practical affairs. Such difficulties of *transfer*, however, should not result in hasty abandonment of mathematical topics without searching examination of their potential values. Rather, we should strive to find those values, and to overcome the difficulties of *transfer* of ideas, not only in our own minds, but surely and most emphatically also in the minds of students. If teachers themselves have such difficulties, it is certain that students will have them, and that they will need patient guidance if they are to integrate what they learn in mathematics with the problems of everyday living.

How shall such thinking be done in ordinary classes? It may be thought that the topics that I have mentioned will occur rare-

ly in ordinary courses, so that little opportunity for training in functional thinking will exist. The fact is that I have mentioned only a few of the numberless instances of functional thinking. There is actually no lack of opportunity for such thinking if it is recognized when it does occur. Thus the simplest conceivable problems of elementary arithmetic all involve such thought. If it be required to find the cost of six apples, for example, the very first question would be "What must we know?" The answer is, "We must know the cost of one apple." This involves the simplest functional thought. It is here what I called *qualitative*. The next question is the *quantitative* one, "If we know the cost of one apple, how do we find the cost of six apples?" The answer is again functional thinking in its simplest phase.

Without abusing your patience by detailed quotation of other examples, I may point out that *every problem stated in Engish* (in arithmetic or in algebra) must of necessity contain and depend upon functional thinking. For in any such problem there must be some quantity whose value is sought, and no problem can exist unless the quantity sought *depends upon* some other quantity (or quantities) given in the problem. The recognition of the dependence of the required quantity upon those that are given, and the formulation of the precise manner in which it depends upon the given ones, is certainly functional thinking, and it is also certainly the most vital thing in the problem. Shall we suppress it—(or slur over it)—or shall we give such thinking the greatest emphasis? If teachers hurry over such considerations with the mistaken idea that the *mathematics* consists in the manipulation of resulting formulas and equations, the real values are lost and mathematics is aborted into meaningless formalism. If every problem stated in English is made the background for functional thinking in the classroom, such thinking will become habitual, and mathematics will become a live thing, of which the formulas are only slaves. Such is indeed, true mathematics. Without such functional thinking, it is a dead form that alone justifies the criticism of those educators who know none of its actual living significance.

I should mention in as great detail the functional thinking in geometry, but time forbids. Let me only correct one possible misimpression. It may be thought that functional thinking occurs only in such mensuration formulas as those for areas and volumes, Consider, however, the very earliest propositions, those about congruence of triangles. We teach that two triangles are

congruent, for example, if three sides of the one are equal, respectively, to the three sides of the other. My own instruction stopped right there, and I was hurried on to other propositions without thinking out the meaning of this one. It is better to stop and to think, for example, that this must mean that any triangle and all its parts, are definitely determined if its sides are known. Did you yourself ever stop to think that this means that each *angle* of a triangle is determined when the sides are given? Of course you realize that formulas exist in trigonometry for this very purpose, but it is not necessary to have the formula; the fact that the angle is determined is a functional thought that should not be passed over slightingly.

This one instance should make clear that functional thinking pervades geometry also. I shall not weary you with other instances, but I may recommend that you yourselves look for functional thinking in every theorem, and in every exercise or problem.

The need for such reality in all mathematics is emphasized for all scientific teaching by a very pungent article by Lancelot Hogben on "The Teaching of Science in the Education of the Citizen." Referring to the somewhat vague report of an English Committee, he says, after an introduction which I will omit, "An illustration is contained in the recent Interim Report of the Science Masters Association on the teaching of General Science. The Committee recommend the place of science in the cultural curriculum because no one

> "can now be considered truly cultured, no one can be considered as having felt the European spirit at its best, if he has never had his imagination stirred by that great adventure of ideas on which we are engaged: the scientific exploration of natural phenomena."

"Ever since the Reformation we have sterilised the teaching of mathematics by adopting Plato's plea for the pursuit of geometry as an aid to spiritual refinement. The world now stands in need of another Reformation and the Committee propose to sterilise the teaching of science by using it to cultivate the European spirit. As a plain citizen I must confess that I do not know what the European *spirit* is, and I am not very excited about the prospect of finding out. I do know what are the outstanding *achievements* of civilization in modern Europe and modern America, and I do know something about how these achievements have affected and have been effected by scientific discoveries. If that is what the Committee are talking about

they should say so, and if I have misinterpreted them they have only themselves to thank. If they wish to convert me they cannot expect me to penetrate a smoke screen of earnest and equivocal phrasemaking. It is their business to state a plain case in plain terms which I, as a plain citizen, can understand.

"As a plain citizen with no interest in watching the antics of the European spirit in the nebulous realm of ideas, the claims of science in education seem to me to rest on very simple and impelling considerations which are easy to grasp and easy to state, In three sentences they are these. The scientific knowledge which is now at the disposal of civilization in modern Europe and America could rid us of war, poverty and disease. If European civilization does not use science to rid itself of them, war will probably destroy our Anglo-American civilization and destroy it irreparably. Knowing how science can be used to advance civilized living and knowing how it may be misused to destroy European civilization is therefore necessary and useful knowledge for a citizen to possess."

It is such vital teaching that I would urge upon you. In mathematics, this can be accomplished only by insistence upon functional thinking in every topic and in every problem stated in English.

We do face the need for such ability to think functionally in a great part of our public life as in our private lives. Thus the public must decide such great questions as those of bond issues and their amortization, installment buying, regulation of usury, proper old-age pension plans. The extreme danger that lies in the advice sometimes given that these things may "be left to the experts" is frighteningly evidenced by the recent elections in which unsound old-age pension schemes were urged upon voters by every political device. It is very clear that the whole public—not merely a few experts—must really think through such functional situations, if we are not to be faced with public bankruptcy and ruin through emotional action by unthinking majorities, precisely because they cannot, or at least do not, think functionally.

In every situation in which quantities occur, from the homely case of the housewife who buys canned goods skillfully packed in tall cans to deceive her if she cannot think functionally, to the grand questions of public weal such as old-age pensions, what is essential is ability to think out the relations between related quantities. Such thinking should be the heart and soul of every

mathematical course. If it is, mathematics contributes, as it of a right ought to do, to public well-being and to private living as well. I would it were so.

Wherever men (or even women) think (by graphs, by experiment, by formulas, or just by common sense) about quantities, wherever the question arises as to "*what*" affects "*this*," and as to just *how* it works, there is functional thinking. To contribute toward better thinking by more people in all such cases is the proper goal of all mathematical teaching in all schools. To deny this is to expose one's own ignorance. To know it, and yet not to practice it, is to pervert mathematical teaching and to be traitor to the educational trust that is the teacher's birthright.

THE TEACHING OF ARITHMETIC

Butler Laughlin
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I. Where in arithmetic should the various processes be taught?

Quantity is a part of the life of the child from the day he is born until his death. Very young children have many quantitative experiences early in life. They are very much interested in the quantitative side of life and manifest this interest very early. There is no reason why rational number should not be taught in the kindergarten and throughout the elementary school. Quantitative thinking is a part of our everyday life. However, most people, even if they live a rich and useful life do not find it necessary to do any computation except in the very simplest arithmetical processes. The computation in which the rules of arithmetic are used seldom occurs. Maturity is far the best teacher of the complexities of quantitative matters.

The first, second, and third grades should do very little teaching of the various arithmetical processes. They should do very little teaching of the various combinations. The time should be spent in dealing with quantitative situations which are part of the social life of the children. They may want to know what three plus two is or they may want to know ten minus seven, but it is not necessary to drill them on the various combinations found in the four fundamental operations. A knowledge of the social phases of arithmetic should be developed so that a need for the combinations is felt. Then they may be taught with greater ease. For the ordinary child, the

fourth grade is the place to teach the various elementary combinations and the simplest operations. Here again the quantitative aspect of life as found in social situations is of much more importance than carrying the combinations to extremes.

The fifth grade is a much better place to work out the four fundamental operations in integers than is the third or fourth grade. Here again the social implications should be kept in the foreground. In teaching the four fundamental operations in integers, make use of simple exercises. In developing an understanding of arithmetic, it is not necessary to get into highly involved problems. Three things should be emphasized in the fifth grade. The principles of arithmetic, accuracy in arithmetic, and the social implications of arithmetic. It is possible that with some fifth grades long division should be postponed until a later time. If one analyzes the psychology of a long division problem he will find a number of complexities beyond the ability of ordinary children.

Make use of the sixth grade to teach the common fractions and their application. In dealing with fractions, only simple, related fractions should be considered as required for all. During the sixth grade a continual review should be carried on in the four fundamental operations in integers to see that nothing that has been taught in previous grades is lost.

Decimals and their application should be taught in the seventh grade. Here again it is not necessary to develop a highly complex notion of decimals. Deal with the more simple social situations. The application of decimals, and quantitative situations dealing with space relations should be taken up in the eighth grade. This, in brief, is where the various processes should be taught.

II. Where should the teaching of formal arithmetic begin?

The word formal has many meanings, and it is doubtful whether any two people will agree just what is meant by formal arithmetic. As has been stated before, quantity is a part of a child's life from the day of his birth. According to my understanding of formal arithmetic, this work should start in the fourth grade. This means that children should be made conscious of the facts of arithmetic and the four fundamental operations in their simplest form. When this is done one might say that formal arithmetic begins. It is very difficult to make any division between the early quantitative work and the more formal phase.

III. What effect will the deferring of arithmetic teaching have upon the high school?

If one has had the experience of checking on the quantitative skill of high school students he will wonder just what was taught in the elementary school. Our present organization of mathematics in the grades forces upon children a number of computational exercises which are far too difficult for them. In the elementary school most teachers have learned that reading readiness is absolutely necessary if normal results are to follow in the field of reading. Someone should work out a method whereby arithmetic readiness might be detected in the early elementary grades. Because children are taught prematurely the various combinations in integers and fractions, many of them get discouraged and inhibitions are set up. Thus they do not learn the things that should be learned. Because of the scattering of teaching efforts in the elementary school, children enter high school with little ability to deal with the social aspects of arithmetic or with arithmetical computations. If the elementary school would attempt fewer things and deal with them on an elementary basis until mastery has been attained, much better products would be turned out. I do not believe that the high school can tell whether arithmetic has been deferred or not because at the present time most children possess scant ability in arithmetic. A change will most likely be for the better. A more simplified course in arithmetic should produce a better high school student. I am not at all afraid of deferring the teaching of arithmetic to the grades suggested in topic one.

IV. How can the placing of the decimal point be taught in division in order to overcome the present difficulties of children in dealing with this process?

Very few people ever have any occasion to add or to subtract common fractions. They always deal with related fractions if with any, unless they are engaged in some of the higher scientific pursuits. Most people occasionally have to deal with the addition and subtraction of decimal fractions. They may have occasion to deal with the multiplication of decimals, but seldom do they have occasion to deal with the division of decimals. I know that I have not lived a very rich life, but in all of my experience, except in the teaching field, I have had no occasion to deal with the division of decimals where I could not have estimated the answer without any knowledge of the division of decimals. My first suggestion is that we teach young people to

make an estimate of the answer without any rule involving division. For most young people this will take care of any need. If the occasion arises where the more brilliant student needs to learn the division of decimals, I recommend the simple method of reducing the divisor to an integer and placing the decimal point in before the operation is started. I have experimented with all the methods of division of decimals and find that this is the only one for which I can vouch. I have seen this tried many times in the elementary school and know that with a successful teacher it will work. This, however, does not mean that a successful teacher cannot use other methods of division of decimals successfully, but I have seen good teachers fail with the other methods and succeed with this one. However, as I have said before, I would not make this a required part of the quantitative situations. Teach division of decimals where the answer, as far as integers is concerned may be found by an estimate. If, in later life, a student in scientific work has reason to learn the division of decimals, let maturity take care of the learning.

MOLECULE, A NEW CHEMISTRY CARD GAME

Sherman Sparks
Mt. Olive, Illinois

Molecule is a new card game that uses chemical atoms, or ions, as units. The object of this game is constructing and naming molecules formed by the combination of two or more of these units.

The deck of cards consists of regular size bridge cards. At either end of the card is printed the symbol of an element or radical. Near the center of the card the valence and atomic weight are printed for the element or radical indicated.

The game is played by dealing seven cards to each player and one face up on the table. The remaining cards are the reserve and are placed on the table face down beside the one card that is face up. The player at the left starts the game either by taking the card exposed or the top card from the reserve. After putting down cards for correct formulas (and naming the molecule) this player discards one card, laying it face up next to the reserve pile. The next player may take up one or more cards from the face up stack providing he can use the lowest card he picks up, or a card from the reserve, plays and discards.

The player who first uses all his cards, in making molecules, and discarding one, closes the hand. Each card left in the hands of the other players counts against them according to the sum of the atomic weights of the cards they hold subtracted from their score which is the sum of the molecular weights they have laid down and declared. Four such hands constitute a game.

If a player lays down the wrong combination or misnames a molecule, he is penalized two times the molecular weight.

The person with the highest score (sum total of all molecular weights for the four hands less penalties) wins the game.

Have you as a Chemistry instructor encountered students that have difficulty in constructing and naming molecules? When writing equations it is essential that the student has this ability.

Some of these slower students have little difficulty in learning the fundamentals of games they enjoy. MOLECULE is a game that is easy to learn while the fundamentals of constructing and naming molecules seems more difficult as we teach them in the classroom.

Students that play MOLECULE prior to their study of molecular construction in the classroom do, as a whole, much better work than the rest of the students in formula writing and the like.

Science Club directors are often at a loss as to what games may be used in their meetings that will be both amusing and educational. MOLECULE is a game that the director will not feel embarrassed to sponsor.

THE SCIENCE CLUB PROGRAM OF THE AMERICAN INSTITUTE

H. H. SHELDON
American Institute, New York City

In any discussion of the science club program of The American Institute of the City of New York, it is first essential that one should answer three questions which are sure to arise. What is The American Institute of the City of New York? What is its purpose in promoting science clubs? What is its procedure in establishing such clubs?

The American Institute was founded in 1828 by a group of far-seeing young men whose purpose was to promote industry, invention and science in this country. At that time practically

all manufactured goods consumed in this country were imported and those few articles of American origin were looked upon as inferior. To encourage domestic industry and invention The Institute inaugurated great commercial fairs, gigantic for their time, where American products were displayed and judged. Such notable inventions as the McCormick Reaper, Singer Sewing Machine, Hoe Press, and the Morse Telegraph were displayed and received awards at American Institute Fairs. There are in existence today more than 75 manufacturing concerns which were awarded Institute medals dating back more than 50 years ago.

Those of you who have seen pictures of the old Crystal Palace in New York will remember something of its size and this will give you a mental picture of the Institute fairs, for it was in this building they were held for many years. It was occupied by an Institute fair at the time of its burning. As time went on this fair was split up into many sections because no place was large enough to house all the divisions of industry. In this way was born the modern industrial shows. The Institute had accomplished its objective. American products were accepted not only in America but throughout the world.

For a time approval of The American Institute was almost a necessity for public acceptance of any manufactured article. Then the Institute sank to a position of relative obscurity. It was not until about twelve years ago that it began once more to rise toward a new crest. As County Fair No. 1 of the State of New York it was necessary for the Institute to hold some form of fair in order to maintain this status and be eligible for such state support as is given to such fairs. Accordingly, the idea arose of holding a science fair for children. The very first of these was an outstanding success, attracting approximately 500 exhibits and for the four days of its duration counting an attendance of approximately 40,000 spectators. Here was a wholly new angle to its original purpose; for while fairs and exhibits in the 19th century served to bring notable inventions before the public, and thus contribute to American commerce and industry, today commerce and industry are the evident outcome of science and engineering.

If we are to continue to hold our position of world supremacy which we have but recently gained in science and engineering, and the resultant commerce and industry, it is clear that this country must do an outstanding job in promoting science edu-

cation. To do this we must recognize genius early. Here the function of the science club is obvious. It would seem further that we must not only prepare our young people to maintain our present leadership but with the youth of Europe spending its time in military training, the next generation of Americans may well be called upon to rebuild the world.

We have also heard much about the necessity for preparation of living in a mechanized world, of the impact of science on society. Here again the science club serves a useful purpose in teaching those students not gifted in science at least the fundamentals of the scientific method of thinking. It is difficult to adjust ourselves to a mechanized world if we have not the slightest idea as to what a mechanized world is. It is for these reasons that the Institute now finds itself interested in promoting science clubs nationally. It is but carrying on the original purpose.

In order to further the interests of science clubs throughout the country, the Institute has prepared considerable printed material such as the booklets *How to Organize a Science Club*, *How to Plan a Science Congress*, and the like. Such material the Institute stands ready to supply to those who have the interest of this work at heart. It also publishes the *Science Observer*, a paper intended for high school students and the *Science Leaflet*, intended for teachers of science and sponsors of science clubs. The Institute is anxious and ready to cooperate with any organization in the United States whose objectives are the same as its own. This naturally includes the various Junior Academies of Science throughout the nation and with some of these the Institute is already actively cooperating. Plans are well under way for similar cooperation with others. It urges also cooperation between science clubs and business organizations and in some cases has attracted active interest on the part of Chambers of Commerce and the like. In such experiences as it has had, it has found that business men generally appreciate the objectives of these clubs once they have been called to their attention. It appears clearly that there is a prospect in the immediate future of interesting business men to become active in the promotion of science in a way that has had no precedent in the past.

In conclusion let me offer to each of you interested in science club work, those helps which we have prepared for the purpose and invite you to discuss with us freely such problems as may arise in the conduct of your science club work.

THE PLIGHT OF HIGH SCHOOL PHYSICS

III. Mismanaged Mathematics

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BACKGROUND

The year is 1872. Famed in education as the year in which the high school was ruled by a court in Michigan to be an integral part of the common school system, in the history of science teaching it is also famed as the year in which physics came of age as a secondary school subject. Courses in Natural Philosophy there had been in America from the establishment of the first public high schools, fifty years previously. Somewhat chaotic as respects both content and aims, in 1867 the newly established U. S. Office of Education had tried to bring order out of the confusion prevailing in these courses. Now, five years later, the subject has become enough like the college physics after which it is being modeled to win the accolade of college recognition. Harvard University has placed the subject on its list of entrance requirements. Standardization to the college pattern has begun.

But justification for the subject is found not to rest entirely upon its college preparatory function. Educational psychology begins to enunciate ever more and more fervently the doctrine of the disciplinary value of education. This is eagerly seized upon as further justification for the subject. *Surely physics has disciplinary value. It's certainly hard enough, and it's not too interesting. An ideal choice.*

With these advantages it might be thought that the entrance of physics into the fold of the academically blest might have been accompanied by somewhat of a disturbance among those subjects already there. But such was not the case then—nor has it been since. Let us look at the entrance requirements of Queens College in New York City. Supported by the taxpayers of New York City and hence independent of tuition money for support, opened in 1937 at a time when educational philosophers such as Kilpatrick are urging the schools to pay increasing attention to the content and methods which are peculiarly those of science—surely here a liberalization of entrance requirements might be expected. Let us examine them. Every student entering the

portals of this institution must present $2\frac{1}{2}$ units of mathematics, 3 units of English (obtained by a study of the subject for four years), 3 units of foreign language and one unit of American history. And how much science? *Not one single unit.*

Progressive Swarthmore exacts a still more rigid requirement —mathematics, English, and history as above, but four years (with a recommendation of five) of a foreign language. And again, no science. Evidently progressivism does not extend to the entrance requirements! Many colleges it is true merely stipulate graduation with a relatively good standing from high schools on the accepted list. But for obvious reasons, the requirements set by colleges with clearly defined entrance requirements, tend to set the pattern for the academic courses in all high schools.

But what has all this to do with the problem of mathematics in relation to high school physics? In a general way, simply this. The subject of physics has developed to be ever more and more (save perhaps in the very rare exception) like its college model. The college subject is a highly mathematical one. The process has been accelerated by the notion of the disciplinary value of the subject. A syllogism suggests itself:

1. The more difficult a subject, the greater its disciplinary value.
2. The mathematical portions of physics are, as a rule, the more difficult, hence
3. The more mathematical the treatment of physics, the greater its disciplinary value.

And perhaps we should add, parenthetically, the greater the approbation of both parents and colleges. Add to this the fact of the prestige enjoyed by courses in algebra, plane geometry, etc., as college admission units, and we have what seems to be a perfectly reasonable explanation for the mathematical character of present-day high school physics.

High Lights

But, more specifically, what conditions are likely to obtain in a physics class during the teaching of mathematical portions of the subject? Let us list these conditions as briefly as possible, with the understanding that, although the three background forces are responsible in greater or lesser degree, the existence of other causes may also be adduced.

1. Mathematical problems are assigned not in relation to the

clarification of physics principles but simply as a task to be performed. Every text contains a goodly number of physics problems requiring numerical answers. In many communities, the texts are paid for by the local Board of Education. Therefore the conscientious teacher often feels that he is somehow not earning his money unless he covers the whole book, problems included. "*For tomorrow work out all the problems on page 246.*"

2. Because of this method of assignment and faulty teaching methods, students fail to grasp the fact that every problem in physics involves the application of one or more physics principles. Possibly as a result, answers are left unlabeled as to unit, or are given with mislabeled units, or with improbable values. *The acceleration is four feet per sec. A force of 10 horse-power was required. The density of the object was 125 grams per cubic centimeter.*

3. Teachers apparently labor under the mistaken notion that they may use mathematics in their physics teaching only if they test upon it. Is this because of a belated survival of the disciplinary objective, or because of the mathematical nature of the C.E.E.B. examinations, or for some combination of these and other reasons? It is hard to say. Certain it is that all too often the answer to the question of whether or not the derivation of a certain formula shall be presented to the class is given in terms of whether the teacher thinks the students can give back the process on a quiz at a later date. If it is believed they can, the derivation is shown. Otherwise it is not.

4. Students, even those with an adequate background of courses in mathematics are often unable to work out rather simple problems in physics. This may be true even though, on many different occasions, they have successfully solved problems in algebra which required exactly the same mathematical manipulation. How difficult the translation from algebra's x to physics' D only the struggling adolescent can tell!

5. Probably because of the condition noted in 4 above, physics teachers are prone to place the blame for their students' difficulty with mathematical problems on the mathematics department. *These students have had all the mathematics courses they need. Yet they can't work these simple problems. What poor teaching they must have had in their math classes!*

6. Enrollment in physics courses is on the down grade. Is this due to the mathematical character of the subject? Probably in some degree. Physics is mathematical—but so is a mathematics

course, surely, and many students who elect mathematics courses in their last years do not elect physics. But have you forgotten the position of prestige held by mathematics courses in the college entrance situation? Why, after all, should a student elect a physics course when a math course is available?

7. Last, and perhaps worst of all, we physics teachers fail to see mathematics in its correct relation to physics. We argue for or against the use of mathematics, basing our arguments on such puerile grounds as that the students can, or cannot, "do" the quantitative work. Rarely do we evidence any real understanding of what mathematics may mean in the high school physics field. Even so great a physicist as Michelson once urged that we teach physics "which avoids as far as possible the use of mathematics of even the most elementary kind." *Mathematize. Demathematize. Why? Oh, because!*

Redress

If we science teachers are to remedy the conditions described above, what is needed, above all, is a new attitude toward the place and function of mathematics in high school physics. Indeed if that new attitude is clearly understood and rigorously applied, that is about all that *is* needed. Let us begin by disassociating from our minds all ideas that certain topics or principles must be presented quantitatively and that certain other topics or principles may be discussed only in qualitative fashion. Instead, concerning each of such topics or principles we ask, "Can the understandings and appreciations which the students of my class may acquire be enhanced in significant amount by the use of mathematics?" If the answer to the question is "Yes" we treat the topic quantitatively even though it has always been treated in a qualitative fashion previously; if "No" we reject the mathematical treatment even though every text in the country gives a full page of numerical problems dealing with subject. Note that the question is individualized. It is *your* class to which the application must be made. No longer do we feel constrained to make mathematical developments because of tradition or to fail to make them because students may not be able to reproduce that mathematical development on a test. We do not base academic achievement ratings directly upon ability to read, a tool skill, however much that ability indirectly may affect the academic standing. Why, then, should we base grades upon the ability to use mathematics, another tool skill?

The danger inherent in the above, apparently simple solution to the problem lies in this very simplicity, a simplicity which conceals the real difficulty of its application. Let us, therefore, attempt concrete illustration of what the application of this principle may mean.[1]

Virtually all high school texts give the formula for the kinetic energy of a moving body. Cast into its most usable form it is $K.E. = \frac{Wv^2}{2g}$. Following this formula there will be a number of problems which ask the student to calculate the kinetic energy of this or that moving object. And, in most cases, that is all. But let us see. Is it all? We take a case in point.

We are using a hammer. On one occasion, as it descends, the head is moving at the rate of 30 feet per second. If the head weighs $\frac{1}{2}$ pound we obtain a value of a trifle over seven by substituting in the formula. Seven what? Why seven foot-pounds. Let us round off our figure at this value.

If we stop our considerations at this point we have succeeded only in helping the student to develop skill in manipulating the formula. Let us, therefore, go further.

This seven foot-pounds is kinetic energy. And energy is defined as "ability to do work." We measure work by the product of force and distance. This means that the seven foot-pounds of energy possessed by the hammer may be used up in driving a small nail a relatively long distance, or a spike a short distance. But enough. The hammer has hit the nail. Measurement reveals that the nail has been driven in a half inch. The energy equation becomes:

$$7 = \text{Force} \times \text{distance};\ 7 = F \times 1/24\ (\tfrac{1}{2}\text{in.} = 1/24\text{ foot}).$$

Solving for F we get $F = 168$ pounds. That is, the average force exerted by the hammer on the nail is that of a stationary weight of 168 pounds.

We pull back the hammer to deliver another blow. The head descends at the same speed as before. But this time, alack, our aim is poor and the hammer hits not the nail of steel but the one on the end of our thumb. Unfortunately, too, that thumb is resting against an unyielding surface. The nail can "give" but a very short distance—about a thirty-second of an inch, that is about 1/400 of a foot. Now our energy equation is

$$7 = F \times 1/400 \text{ and } F = 2800 \text{ pounds.}$$

A force of nearly a *ton and a half* has momentarily pressed against our nail! Is it any wonder that a few days or weeks later we lose the nail?

Now this is only one series of mathematical experiences but it should be apparent that we are commencing to build up understandings concerning the meaning of kinetic energy and work and of the interconvertibility of energy forms. Note that in

[1] A more detailed discussion of the use of mathematics in high school physics, which includes a goodly number of illustrations of how mathematics may enrich the teaching of physics is given in the following reference: Brown, H. Emmett. "Mathematics in Physics." *Mathematics in Modern Life*, 6th Yearbook. The National Council of Teachers of Mathematics, pp. 136–164. Teachers College, Bureau of Publications. New York. 1931.

order to achieve our purposes we went beyond the treatment accorded the topic in most texts. Far better to treat kinetic energy in a purely descriptive fashion than to employ the usual incomplete mathematical development.

Space precludes a long list of illustrations but let us indulge in one more.

Newton's Third Law of Motion is stated simply as: "To every action there is an equal and opposite reaction." Now just what does this mean? Let us consider the case in which a cartridge is being shot from a rifle. Visualize the shell traveling down the rifle barrel impelled by the force of expanding gases. Evidently the force exerted on the shell (F_s) equals the force exerted backwards on the rifle (F_r); similarly the length of time during which this force is being exerted is the same in the two cases. We make a table showing the equalities obtained so far:

Shell	Rifle
F_s	F_r
t_s	t_r

and of course

$F_s \times t_s$	$F_r \times t_r$

From Newton's Second Law of Motion we know that, in general, $F = Wa/g$. We substitute for F, using appropriate subscripts, in the above expressions and obtain two new expressions to insert in our table. They are $W_s a_s t_s/g$ and $W_r a_r t_r/g$. But from the work on accelerated motion we know that $v = at$. Again substituting with appropriate subscripts, we obtain two more equal expressions which, when inserted in the table makes it appear as follows:

Shell	Rifle
F_s	F_r
t_s	t_r
$F_s \times t_s$	$F_r \times t_r$
$\dfrac{W_s \times v_s}{g}$	$\dfrac{W_r \times v_r}{g}$

To these last two expressions, definite names have been given. They are *impulse* (Ft) and momentum (Wv/g). In this case we may therefore replace the less precise terms "action" and "reaction" by the more definite "momentum of the shell," "momentum of the rifle" and "impulse of the shell," "impulse of the rifle."

In conjunction with the work done previously we can now refine notions of kinetic energy and momentum, terms which are commonly confused. In the previous problem we equated energy to work ($F \times d$). Now we equate momentum to impulse ($F \times t$).

In order to make the point clear let us take the case of a boy sliding down hill. Sled and boy weigh 120 pounds. At the bottom of the hill they are moving at the rate of 16 feet per second. Calculating, we learn that the kinetic energy is 480 foot-pounds; momentum has the numerical value of 60 pound-seconds. Let us suppose that the force of friction on the level surface onto which the sled now goes is six pounds. How far will the boy go? Why 80 feet. ($480 = 6 \times d$; $d = 80$). For how long a time will he move? Why, 10 seconds. ($60 = 6 \times t$; $t = 10$).

Thus we see that we can enrich our notions of kinetic energy and momentum by the following:

Kinetic energy is the measure of the DISTANCE *that a moving body will continue to move against a constant retarding force. Momentum is the measure of the* TIME *that a moving body will continue to move against a constant retarding force.*

By our mathematical development we have at once made the Third Law of Motion more understandable and have also further clarified our notions of momentum and kinetic energy.

The treatment suggested in these two illustrations may (or may not) seem attractive and worthy of adoption in your classes. But let us not be precipitate. The question as to whether or not you use it in your own classes will depend not upon your judgment of your student's ability to understand the mathematical development but upon the importance you attach to the concepts of *work, kinetic energy, the Laws of Motion, momentum,* etc. If you think of them as relatively unimportant, you may not teach them at all, or you may content yourself with a qualitative development. For any really *thorough* understanding the mathematical development is probably necessary.

Techniques

Finally let us briefly enumerate some of the teaching techniques by means of which the values inherent in the use of mathematics in high school physics may the more completely be realized.

1. So that students may acquire confidence in their own ability to handle the mathematical portions of the subject, there should be a greater use of simple problems with easy figures. The trouble with these portions has been that even those pupils who have an adequate mastery of mathematical skills have feelings of inferiority and unfamiliarity when they come to use these skills in the new field.

2. Almost as a corollary to number 1 is the fact that the teacher is responsible for making the connection and interpretation between the physics problems and the mathematical expressions whose solution will give the desired answer. This process must be repeated on many occasions throughout the year.

3. The numerical problems which the pupil is asked to solve should be as meaningful and challenging as possible. Problems based on his own experience, on work done in the laboratory, on interesting situations which are described in the problem, are of the type suggested.

4. A valuable procedure is to ask students to indicate, before beginning the solution of the problem, the principle or principles which are involved therein.

5. We should give our students numerous opportunities to generalize from data—to take a table of data, for example, and to state the relationship that probably exists between the quantities therein. This technique is of considerable value in helping students to see the basis for certain of the formulae that we employ.

6. In solving problems, students should be encouraged, and in some cases required, to use the "think-through" method. For example, the problem: "How far will a freely falling body move in 3 seconds, ignoring friction?", can easily be solved without the use of the usual formula $S=\frac{1}{2}at^2$. The student sees that the speed at the end of three seconds will be 3×32 or 96 feet per second. Thus, the average speed is 48 feet per second. To get the distance we multiply average speed by time (48×3) giving 144 feet. The formula can be introduced later, if desired, as a generalization upon experience.

7. A technique that is of considerable value in helping to prevent both the substitution of quantities with the wrong dimensions (as *miles per hour* instead of *feet per second*), as well as the mislabeling of answers is to substitute units directly in formulas. Thus the formula for centrifugal force is $\text{C.F.}=\frac{Wv^2}{gr}$. Substituting a correct set of units we get

$$\text{C.F.}=\frac{\text{lbs.}\times\left(\frac{\text{feet}}{\text{sec.}}\right)^2}{\frac{\text{feet}}{\text{sec}^2}\times\text{feet}}.$$

It is evident that everything cancels out except the single unit, lbs. Thus we see that the answer to the problems on centrifugal force should be expressed in pounds if the weight is expressed in pounds. However we do not need to limit ourselves to this unit. An examination of the above substitution will reveal that the weight may be expressed in any unit whatsoever whereupon the answer will be expressed in the same unit. Examination also shows the necessity for the three-quantities v, g, and r, to be expressed in consistent units e.g. feet per sec, feet per sec^2, feet; cm per sec, cm per sec^2, cm. No "cross-breeding" of units is

tolerated. In order for the method to have the maximum value we must require that the answers to all problems be clearly labeled as to units.

8. The physics teacher must not hesitate to teach the mathematical skills that are needed in the solution of such numerical problems as he may wish to use. A helpful procedure is to inventory the class skills by giving such a test as the Kilzer-Kirby Test (Public School Publishing Co., Bloomington, Ill.) which will reveal deficiencies that may exist. Or the teacher may easily prepare short inventory tests which are given just prior to the class work on a new type of mathematical problem. Thus, before problems on pressure-volume relationships are assigned, a short test on the solving of proportions may be given. Needless to say no grades are given on such tests but the results are used to see if the class, or certain individuals therein, need review on this skill.

9. Probably as a special case under the preceding point, but of sufficient importance to deserve a separate listing, is the technique of teaching students that a proportion of the form $A_1/A_2=B_1/B_2$ is the mathematical equivalent of the physical statement: "A varies directly as B" and that the proportion $A_1/A_2=B_2/B_1$ is the equivalent of "A varies inversely as B."

10. Finally, and by way of summary, the teacher must hold constantly before him some clear concept of the function of mathematics in high school physics—the one that is enunciated here, or one of his own devising. He must hold clearly to the precept, "No mathematics for reasons of tradition or expediency alone, but mathematics in liberal amount to enrich understanding and appreciation of topics and principles that are adjudged, on what are believed to be valid criteria, worthy of inclusion in a modern physics course."

"LET'S FORM A JUNIOR AUDUBON CLUB"

This is an idea for boys and girls interested in watching birds this spring and in finding out how they live. Junior Audubon Clubs are sponsored by the National Association of Audubon Societies, an organization whose purposed is the protection of all forms of wildlife. A special endowment enables the Association to furnish its Junior Members with interesting material at far below cost. TEN or more children may band together to form a club in a school, Scout troop, camp or in the home neighborhood. Club dues are ten cents a member each year. Each Junior Member receives a bird button and six four-page bird leaflets with bird color plates and outline drawings to color. For other details and registration form, write to the Junior Secretary, National Association of Audubon Societies, 1006 Fifth Avenue, New York, N. Y.

WHY NOT USE THE ENGLISH SYSTEM IN SCIENCE?

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Some years ago when most secondary schools were college preparatory in nature, it was a foregone conclusion that the metric system should be learned as a requisite of most science courses. Those who took college science made use of this knowledge and probably were definitely converted to the system, that is, if their life work happened to fall along scientific lines. The others promptly forgot all about it and the time spent in mastering the system was needlessly wasted. Everyone realizes that the metric system has many advantages over the English system and all wish that it could be used universally throughout the United States. However the enormous investment in weighing and measuring devices that would be made obsolete by such a change will be a dominant factor in preventing any such change in the near future. Then too there is the problem of persuading one hundred and ten million people to adapt themselves to a wholly new system of measurements. These facts should be kept in mind when deciding which system shall be taught and to whom it shall be taught. While there is little doubt that the metric system simplifies the computation in chemistry and physics, I question the necessity of teaching it in general science or even in biology. Certainly it is better to use the less exact English units than it is to use metric units without any explanation of their significance which is all too often the case in the earlier sciences. Of course metric units and devices are used because there is no satisfactory or comparable English unit. As a result we use the English system part of the time and the metric system the rest of the time. This mixture of the systems is particularly confusing to students in the practical or consumer science courses. Imagine the perplexity of the student who is studying the heating value of fuel oil. The heat equivalent of oil is measured in B.T.U.'s per pound. This oil is purchased by the gallon, and if he wishes to convert gallons to pounds he uses specific weight and compares it with the weight of a gallon of water which is measured in pounds per cubic foot. To further complicate matters when he studies foods he finds their heat value measured in calories. Is it logical for us to expect these students to be able to apply their knowledge of science to

an everyday life situation or problem? We who have had many years of experience in dealing with these conversions acquire a proficiency which makes us prone to forget our own struggles in mastering them and to underestimate their difficulty. If we wish to make our science courses truly functional and reach the population as a whole it will be necessary for us to simplify our terminology and do our quantitative work in the units that are now being used by the general public and that means the English system.

As we have previously mentioned there is no satisfactory and exact volume-weight relationship in the English system. Let us make a brief comparison of the units of the two systems.

Metric;

Unit of Volume	cc	Liter	The units of weight being defined
Unit of Weight	1 gm	1 kg	in terms of the unit of volume.

English;

Unit of Volume	?	Fluid ounce	?	Pint	Quart	Cubic Foot
Unit of Weight	Ounce	?	Pound	?	?	62.4 lbs.

It is not necessary to point out the advantages of the volume-weight relationship in the metric system and the disadvantage of the lack of a comparable relationship in the English system. The pint of water weighs nearly a pound but is not sufficiently exact for accurate measurement. The present multiplicity of values of the fluid ounce is a sad commentary on our lack of organization in our own system of measures. It is also unfortunate that the only usable English unit of volume is named in terms of a unit of weight to which it is not equivalent.

As a possible solution to this problem I propose that we develop a standard unit of volume in the English system based on the VOLUME occupied by a POUND of Water either at 70°F. or 20°C. What this unit shall be called is not important now but it should have a different name than its equivalent unit of weight. The table below illustrates a proposed system of units. The names given to the units are simply for the purpose of designation. In use; A "unit" of water weighs 1 ounce and a "unit"

Unit of Weight	Pound	Ounce	1/10 ounce
Unit of Volume	"Menser"	"Unit"	"Decit"
Cubic equivalent	28.6 cu.in.	1.73 cu.in.	0.173 cu.in.
Metric equivalent	453.6 cc.	28.35 cc.	2.83 cc.
Heat equivalent	1 B.T.U.	.062 B.T.U.	.0062 B.T.U.
	252 cal.	15.7 cal.	1.57 cal.

of alcohol would weigh about .8 ounce. The density would be .8 ounce per "unit" and the specific weight .8 as in the metric

system. The volume of solids measured in cubic inches can be reduced to units by dividing the volume by 1.73. The specific weight may then be determined by dividing the weight in ounces by the volume in "units." A glass cylinder calibrated in "units" would be a most useful device as it would replace the familiar metric graduate. Such a system of units would be a boon to thousands of amateur photographers who could then make up their developers from stock solutions obtaining the required amounts of materials by volume rather than weight. The cost of such a graduated cylinder is only a fraction of the price of a good scale and a set of weights. If such a "unit" could be developed it might be possible to nationally redefine the pint in terms of the pound of water and the quart as two pints. The change in volume would not be noticed by the general public although it is quite evident in the laboratory.

I would not be so presumptuous as to suggest that these units be adopted without a great deal of research as to their possible use in business as well as scientific fields. I suggest them rather in the hope that others will become interested in this problem and that eventually there will evolve a more simplified and usable system of English system measurement, a system that will conveniently lend itself to application in the laboratory as well as in business and industry. With such a tool the findings and techniques of science can be applied by the whole population in their daily lives.

A NEW PROOF OF THE PYTHAGOREAN THEOREM

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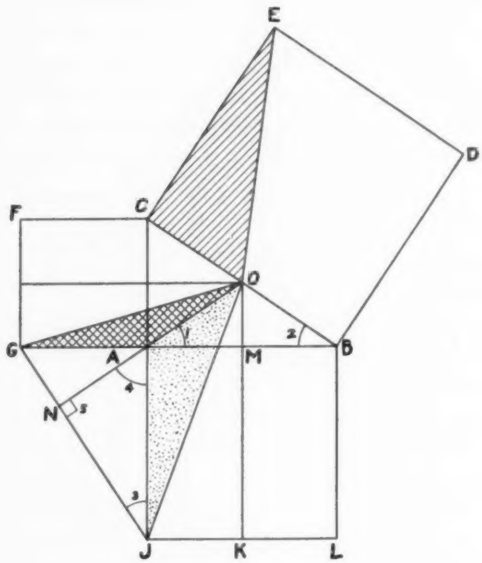
[Math. Ed. The following proof of the Theorem of Pythagoras was made by Ann Condit while a sophomore in the Central Junior-Senior High School, South Bend, Indiana. She is now a senior in the same school. Mr. Wilson Thornton, Head of the Department of Mathematics, is her instructor.

The proof is apparently new. The uniqueness of the proof consists in taking the midpoint of the hypotenuse as the starting point of all auxiliary lines.]

Draw squares $CAGF$, $AJLB$, $BDEC$ on the sides of the triangle. From O, the mid-point of the hypotenuse CB, draw ON through A to the line GJ, which joins the corners of the two squares. Construct $\triangle$s OAJ, OAG, OCE by lines from O to the

corners of the squares. (OK is constructed perpendicular to JL.)



Area $\triangle OAJ=\dfrac{AJ\times AM}{2}$

$AM=MB$

$\triangle OAJ=\dfrac{\square AJLB}{4}$

Similarly $\triangle OAG=\dfrac{\square AGFC}{4}$ and $\triangle OCE=\dfrac{\square CBDE}{4}$

$\angle 1=\angle 2=\angle 3$

$\angle 1+\angle 4=90°;\quad \angle 4+\angle 3=90°;\quad \angle 5=90°.$

Area $\triangle OAJ+\triangle OAG=\dfrac{OA\times JN}{2}+\dfrac{OA\times GN}{2}$

$=\dfrac{OA(JN+GN)}{2}=\dfrac{OA\times GJ}{2}$

Area $\triangle OCE=\dfrac{OC\times EC}{2}=\dfrac{OC\times CB}{2}$

Area $\triangle OAJ+\triangle OAG=\triangle OCE$

$\square AJLB+\square AGFC=\square CBDE$

HIGH LIGHTS OF THE APRIL SKIES

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[EDITOR'S NOTE: This is the sixth of a series of articles on popular astronomy. If these articles are saved from month to month, they make a convenient handbook of astronomical facts and classroom activities for elementary and secondary school teachers.]

Now that the earth has passed the time of the vernal equinox and Easter is past, and spring, with the "April showers" is well under way, the feature of the month is the annular eclipse of the sun. The eclipse will be visible in the United States on Sunday, April 7th. The approximate time of the beginning in four time belts is as follows: New York City 3:50 P.M.; Chicago, 2:34 P.M.; Denver, 1:00 P.M.; and Los Angeles, 11:25 A.M. The eclipse will last about one and one-half hours in each place.

I. CONSTELLATIONS AND STARS

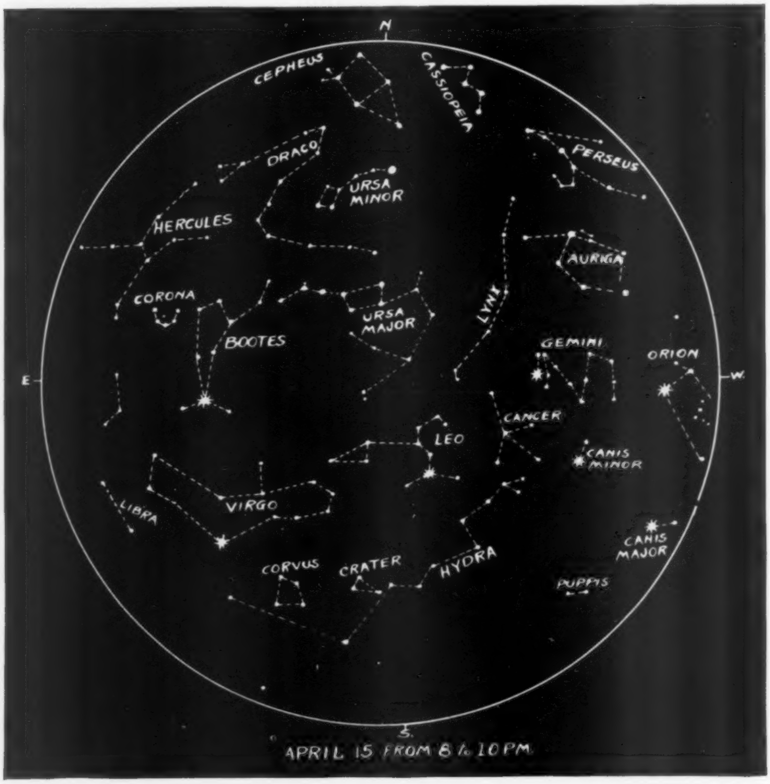
This month several new constellations rise over the eastern horizon and appear higher in the sky each evening as April marches on. Not only do the constellations circle the sky once a day, but they rise two hours earlier each month. The reason for this is that the day is divided into twenty-four hours, and the year into twelve months. As the constellations appear to make one circuit of the heavens in one year, hence they must rise two hours earlier each month.

The constellation *Virgo* is now well up in the eastern sky. Her bright star *Spica* is a brilliant object. In the south, with its head just below *Cancer* and extending in a long line south of *Corvus* and *Crater*, is the constellation *Hydra*. This group consists of a stream of stars about 105 degrees in length, somewhat suggesting the water snake from which it gets its name. In the eastern end is the widely known variable star *R Hydra* which varies from a magnitude of 3.5 to 10 and back again in about 425 days.

The *Big Dipper* in the constellation *Ursa Major* is a group of stars with which everyone should become familiar. Of all the constellations, this one is perhaps the best known. The two stars on its western end are known as "the pointers" and point to *Polaris*, the North Star. The constellation *Ursa Major* is known as the "*Big Bear*" and includes many more stars than the seven that make up the figure of the "*Big Dipper*." It is significant to note that the stars forming the "*Big Dipper*" are circumpolar to observers north of 40 degrees latitude. North of this point these stars never set, but seem to travel around the pole star in a large circular orbit once in twenty-four hours, due to the earth's rotation. Other circumpolar constellations to northern observers are *Draco*, *Cepheus*, *Cassiopeia* and *Ursa Minor*, the "Little Dipper."

The star in the center of the handle of the "*Big Dipper*" is called *Mizar*, and was the first to be discovered and mentioned as a double star. Close beside *Mizar* is a neighbor *Alcor*, a star of the fourth magnitude. On a clear night *Alcor* is not difficult to see, yet somehow, locating it has always been a home-spun test for vision. Some say the Indians used it for this purpose.

The constellations *Bootes*, and *Corona* (*Borealis*) appear over the east later in the evening, with *Hercules* and the other summer constellations



congregating along the eastern horizon to begin anew their yearly spectacle *Corona* (*Borealis*) contains the very famous variable star *R Corona* that can be observed and studied with the aid of only a small telescope.

II. The Planets

Venus is the undisputed queen of the skies this month. She shines with unsurpassed brilliance high in the western sky. On April 17th *Venus* reaches its greatest eastern elongation, which means that from that time on *Venus* will appear lower in the western sky each evening. With a small telescope *Venus* will be observed to be in phase. On April 1st 59 per cent

of the disc will be visible. On the 17th of April 50 per cent can be seen, when the planet changes from gibbous to crescent phase. May 1st only 43 per cent of its disc is visible to us, while on June 1st we can see only 18 per cent. Nevertheless it appears brighter each evening until the 20th of May, since its rapid approach to us more than makes up for its diminishing crescent.

Mars is still in the evening sky, and on the 11th of April arrives at a conjunction with *Venus*. They will appear close together in the western sky with *Mars* $2°11''$ south. *Jupiter* and *Saturn* have slowly disappeared below the western horizon and are in conjunction with the sun April 11th and April 24th, when they pass to the morning sky.

III. Class Room Activities

It is easy to understand why the ancient people thought that the sun, moon and stars moved around the earth. They had no methods of proving otherwise. There are even groups of people today who insist this is the case. It was not until the last century that it was possible to adduce proofs in the laboratory independent of the observations of the heavenly bodies that this was a false notion. A simple experiment often employed to demonstrate the rotation of the earth is known as the Foucalt pendulum test. This can be carried on in most any classroom.[1]

First obtain a heavy metal ball, weighing 15 pounds or more, and, with a thin piece of piano wire, suspend it from the ceiling of the room so that the ball just clears the floor. Draw the ball to one side with a piece of cotton string or fish line, fastened to a hook in the wall in such a way that when the string is severed the ball will be free to swing. When this is done it is important to allow ten or fifteen minutes to elapse to allow the heavy ball to come to a complete rest. In the meantime, draw a chalk line on the floor, marking the apparent path the ball will follow on its initial swing. Touch a lighted match to the cord close to the iron ball. When it is burned through, the ball will begin to swing. In a few minutes it will be noticed that the ball does not swing directly over the chalk line previously drawn on the floor. The floor of the class room seems to turn while the ball swings in the same plane. The longer the ball continues to swing, the more apparent this fact becomes. At the latitude of New York the rate of motion of the floor will be about 10 degrees an hour. At the equator there will be no tendency for the floor to shift, but at the north pole the floor will make one complete revolution in 24 hours. The direction of motion is of course opposite in the southern hemisphere. The hourly deviation of the pendulum for any locality will be equal to 15 degrees multiplied by the sine of the latitude.

This experiment is always of great interest to the class, teachers and visitors. It gives one a queer feeling of insecurity to observe the floor seeming to move under one's feet.

[1] A complete outfit can be purchased for as little as $15.00 from *Central Scientific Company*, 1700 Irving Park Boulevard, Chicago, Illinois. See School Science and Mathematics, November, 1939, back cover.

PROBLEM DEPARTMENT

CONDUCTED BY G. H. JAMISON
State Teachers College, Kirksville, Mo.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The Editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to G. H. Jamison, State Teachers College, Kirksville, Missouri.

SOLUTIONS AND PROBLEMS

NOTE. Persons sending in solutions and submitting problems for solution should observe the following instructions.

1. Drawings in India ink should be on a separate page from the solutions.

2. Give the solution to the problem which you propose if you have one and also the source and any known references to it.

3. In general when several solutions are correct, the one submitted in the best form will be used.

LATE SOLUTIONS

1625, 8, 33, 39. *Walter R. Warne.*

1634, 6, 7, 9. *Charles W. Trigg.*

1640. *Proposed by John P. Hoyt, Cornwall, N. Y.*

If a trapezium $ABCD$ is circumscribed about a circle of radius r, and if the equal tangents are designed by a, b, c, d, respectively, show by elementary geometry that the square of the radius is equal to

$$\frac{abc+acd+adb+bcd}{a+b+c+d}$$

First Solution by John F. Wagner, Lewis Institute, Chicago

Let the central angles subtended by a, b, c, d be $\alpha, \beta, \gamma, \delta$ respectively. Then $2\alpha+2\beta+2\gamma+2\delta=360$ or, $\alpha+\beta=180-\gamma-\delta$. And by taking the tangent of both sides, $\tan(\alpha+\beta)=\tan(180-\gamma-\delta)=-\tan(\gamma+\delta)$. Expanding.

$$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=-\frac{\tan\gamma+\tan\delta}{1-\tan\gamma\tan\delta}. \tag{1}$$

But since $\tan\alpha=a/r$, $\tan\beta=b/r$, $\tan\gamma=c/r$, and $\tan\delta=d/r$, (I) gives

$$\frac{a/r+b/r}{1-ab/r^2}=-\frac{c/r+d/r}{1-cd/r^2}.$$

When this is solved for r, we find

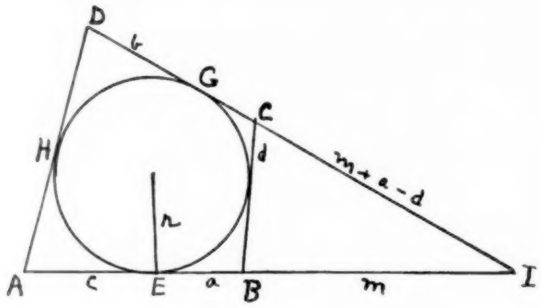
$$r^2=\frac{abc+abd+acd+bcd}{a+b+c+d}.$$

Second Solution by Proposer

Let AB and DC produced to meet at I. Let $BI=m$, then $IG=IE$ and $IC=m+a-d$. The circle is inscribed in triangle AID and escribed to BIC.

Hence (1) $\triangle AID=r(m+a+b+c)$ from the theorem: The area of a triangle equals the inradius times half the perimeter.

(2) $\triangle IBC=r(m-d)$. From the Theorem: The area of a triangle equals the ex-radius times the semi-perimeter minus the side to which the circle is relative.



$$\triangle IBC/\triangle AID=\frac{m(m+a-d)}{(m+a+c)(m+a+b)}. \tag{3}$$

Two triangles having an angle in common are to each other as the products of the sides including the common angle.

Substituting from (1) and (2)

$$\frac{r(m-d)}{r(m+a+b+c)}=\frac{m(m+a-d)}{(m+a+c)(m+a+b)}. \tag{4}$$

Solving this equation for m,

$$m=\frac{(a+c)(a+b)d}{bc-ad}. \tag{5}$$

Using the value of r, the in-radius of the triangle:

$$r=\sqrt{\frac{(s-a)(s-b)(s-c)}{s}},$$

where s is half the perimeter of the triangle, we obtain where $s=m+a+b+c$,

$$r^2=\frac{bc(m+a)}{m+a+b+c}. \tag{6}$$

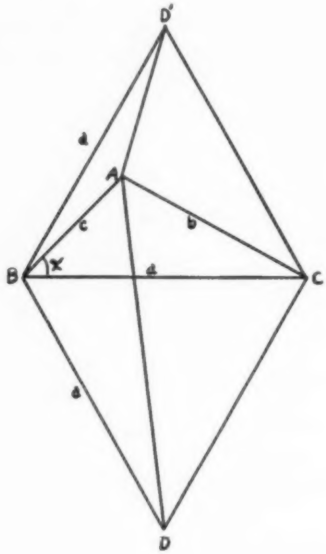
Substituting the value of m from (5) in (6), we obtain

$$r^2=\frac{abc+acd+abd+bcd}{a+b+c+d}.$$

Solutions were also offered by M. Kirk, West Chester, Pa., Sidney V. Soanes, Toronto, Ontario, Canada, C. W. Trigg, Los Angeles City College.

1641. *Proposed by W. R. Warne, Rochester, N. Y.*

On the side BC of a triangle ABC, two equilateral triangles BCD and BCD' are constructed. Prove that $\overline{AD}^2+\overline{AD'}^2=a^2+b^2+c^2$.



Solution by H. R. Shelden, Cicero, Ind.

Since the triangles BCD and BCD' are equilateral and constructed on side a of triangle ABC, BD and BD' are equal to a.

By Law of Cosines,

$$\overline{AD}^2=a^2+c^2-2ac\cos(60°+x) \qquad (1)$$

$$\overline{AD'}^2=a^2+c^2-2ac\cos(60°-x) \qquad (2)$$

$$b^2=a^2+c^2-2ac\cos x. \qquad (3)$$

$$\cos(60°+x)=\cos 60°\cos x-\sin 60°\sin x$$

$$\cos(60°-x)=\cos 60°\cos x+\sin 60°\sin x.$$

Substituting in (1) and (2) and adding,

$$\overline{AD}^2+\overline{AD'}^2=2a^2+2c^2-4ac\cos 60°\cos x. \qquad (4)$$

Since $\cos 60°=0.5$, equation (4) becomes,

$$\overline{AD}^2+\overline{AD'}^2=2a^2+2c^2-2ac\cos x. \qquad (5)$$

Adding equations (3) and (5) give,

$$\overline{AD}^2+\overline{AD'}^2+a^2+c^2-2ac\cos x=2a^2+2c^2-2ac\cos x+b^2.$$

Transposing and simplifying,

$$\overline{AD}^2+\overline{AD'}^2=a^2+b^2+c^2.$$

Solutions were also offered by John P. Hoyt, Cornwall, N. Y., C. W. Trigg, Los Angeles City College, A. MacNeish, Chicago, Ill., John F.

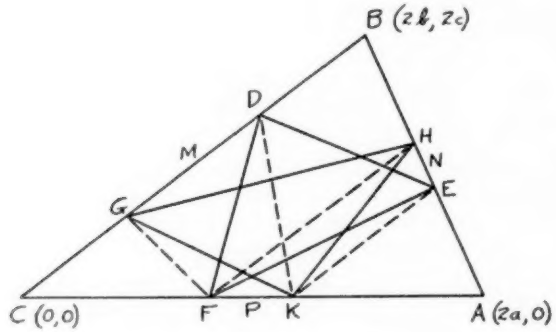
Wagner, Chicago, Arthur Danzl, Collegeville, Minn., Aaron Buchman, Buffalo, N. Y., Oakley T. Herrell, Des Plaines, Ill., B. Felix John, Pittsburgh, Pa., George Jo Ross, Brooklyn, N. Y., M. Kirk, West Chester, Pa., Paul C. Overstreet, Wilmore, Ky., and also by the proposer.

1642. *Proposed by Alvert Hansen, St. Paul, Minn.*

Two triangles whose vertices lie on the side of a given triangle at equal distances from their mid-points are equal in area.

Solution by C. W. Trigg, Los Angeles City College

Let the vertices of triangle ABC be $A(2a, 0)$, $B(2b, 2c)$ and $C(0, 0)$. Then the midpoints of the sides are $M(b, c)$, $N([a+b], c)$, and $P(a, 0)$. If points on the sides are chosen so that each pair is equidistant from its associated midpoint, then these points will be, $G[(b-m), (c-n)]$, $D[(b+m), (c+n)]$, $H[(a+b-p), (c+q)]$, $E[(a+b+p), (c-q)]$, $K[(a+k), 0]$, and $F[(a-k), 0]$. Here, $m:n::b:c$ and $p:q::(a-b):c$, so $mc=bn$ and $q(a-b)=cp$.



If the alternate points are chosen as the vertices of triangles, then twice the area of $\triangle DEF$ is

$$\begin{vmatrix} (b+m) & (c+n) & 1 \\ (a+b+p) & (c-q) & 1 \\ (a-k) & 0 & 1 \end{vmatrix} = mc-bn+q(a-b)-cp-k(n+q)-mq-ac-pn$$

$$= -[k(n+q)+mq+pn+ac].$$

Twice the area of triangle GHK is

$$\begin{vmatrix} (b-m) & (c-n) & 1 \\ (a+b-p) & (c-q) & 1 \\ (a+k) & 0 & 1 \end{vmatrix},$$

which upon expansion and reduction yields the same value as that secured for $2\triangle DEF$.

The other possible choice of vertices of the two triangles is exemplified by the triangles DEK and GHF. For this case we have

$$\begin{vmatrix} (b+m) & (c+n) & 1 \\ (a+b+p) & (c-q) & 1 \\ (a+k) & (0) & 1 \end{vmatrix} = k(n+q)-(mq+pn+ac) = \begin{vmatrix} (b-m) & (c-n) & 1 \\ (a+b-p) & (c+q) & 1 \\ (a-k) & 0 & 1 \end{vmatrix}$$

Hence the theorem is true in general, and the points may lie on the sides or the sides produced.

Solutions were also offered by Walter R. Warne, Rochester, N. Y., John P. Hoyt, Cornwall, N. Y., A. MacNeish, Chicago, Ill.

1643. *Proposed by Stephen Droenus, Willard, N. Y.*

If n is a positive integer greater than one, show that $n^5-5n^3+60n^2-56n$ is a multiple of 120.

Solution by D. G. Burkley, Upper Canada College, Toronto, Canada

$$\begin{aligned} &n^5-5n^3+60n^2-56n\\ &\quad=n(n-1)(n^3+n^2-4n+56)\\ &\quad=n(n-1)[(n-2)(n+1)(n+2)+60]\\ &\quad=60n(n-1)+(n-2)(n-1)(n)(n+1)(n+2). \end{aligned}$$

Since minimum value of n is 2, $60n(n-1)$ is a multiple of 120 as factors n, $n-1$, are consecutive and so one of them must be even. $(n-2)(n-1)(n)(n+1)(n+2)$ is the product of 5 consecutive integers. The product of any five consecutive integers can be reduced to a product containing 2, 3, 4, 5 as factors. $2\times3\times4\times5=120$

$\therefore (n-2)(n-1)(n)(n+1)(n+2)$ is a multiple of 120 and
$60n(n-1)+(n-2)(n-1)(n)(n+1)(n+2)$ is a multiple of 120 or
$n^5-5n^3+60n^2-56n$ is a multiple of 120.

Solutions were also offered by Aaron Buchman, Buffalo, N. Y., George J. Ross, Brooklyn, N. Y., M. Kirk, West Chester, Pa., C. W. Trigg, Los Angeles, John F. Wagner, Chicago, Ill.

1644. *Proposed by S. W. Hall, Newton, Kan.*

Without the use of partial derivatives show that for a triangle inscribed in a circle to have a maximum area it must be equilateral.

First Solution by Alan Wayne, New York

By the following simple Lemma, itself easily proved, the solution can be accomplished by elementary geometry: The length of the perpendicular to a chord from the midpoint of its intercepted arc is greater than the length of the perpendicular to the chord from any other point of the arc.

Let any $\triangle ABC$ be inscribed in circle 0. Construct isosceles $\triangle DBC$ with $DB=DC$, and equilateral $\triangle DEF$. Either $BC>DE$, $BC=DE$, or $BC \angle DE$. Assume $BC \angle DE$.

Draw CE. Since $\triangle DEG \sim \triangle BCG$,

$$\triangle DEG:\triangle BCG=\overline{DE}^2:\overline{BC}^2. \quad \text{But} \quad DE>BC. \quad \therefore \triangle DEG>\triangle BCG.$$

Hence $\triangle DEC>\triangle DBC$. But to DE, altitude from $F>$altitude from C, by Lemma, hence $\triangle DEF>\triangle DEC$. Likewise, $\triangle DBC>\triangle ABC$. Therefore $\triangle DEF>\triangle ABC$.

If $BC>DE$, the proof follows in essentially the same manner from the new diagram. If $BC=DE$, the proof follows at once from the Lemma. Q.E.D.

Second Solution by Malcolm Kirk, West Chester, Pa.

Given a variable triangle ABC inscribed in a circle of radius R. Denoting the variable area by M.

By use of trigonometry the following relations are established:

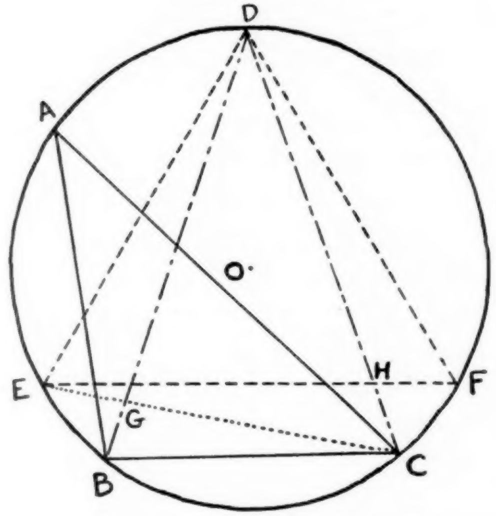
(1) $$M = R^2/2(\sin 2A + \sin 2B + \sin 2C)$$

(2) $$M = R^2[\sin (A+B)\{\cos (A-B) - \cos (A+B)\}].$$

The largest value of M is seen to be when the cosine function is largest. This occurs when the angle $A-B=0$. Hence $A=B$ is a necessary condition for a maximum area. From (2)

$$M = R^2(\sin 2A - \sin 2A \cos 2A)$$

(3) $$dM/dA = -R^2(2\cos^2 2A - \cos 2A - 1).$$



The solution $A=0$ for $dM/dA=0$ is necessarily rejected. Another solution is $A=60°$, from which $B=C=60°$.

Solutions were also offered by J. M. Maxey, Wilmore, Ky., Paul C. Overstreet, Wilmore, Ky., Arthur Danzl, Collegeville, Minn., Cecil B. Read, Wichita, Kan., C. W. Trigg, Los Angeles, John F. Wagner, Chicago, Ill., Carl Noble, University of Iowa, Iowa City.

1645. *Proposed by Garrett Freeleigh, Watertown, N. Y.*

If S is the area of the triangle ABC and S' that of the pedal triangle whose sides are a', b', c', show, that $S:S' = (abc/2a'b'c')$.

Solution by R. W. Laidlaw, Upper Canada College, Toronto

In $\triangle ABC$, $R = abc/4S$ and in $\triangle A'B'C'$, $R = a'b'c'/4S'$. From these relations

$$S = abc/4R$$
$$S' = a'b'c'/4R'$$
$$S:S' = abc\cdot 4R'/a'b'c'\cdot 4R$$

Now R' = radius of the nine point circle

$= \frac{1}{2}$ radius of circumcircle of $\triangle ABC$

$\therefore S:S' = abcR'/a'b'c'\cdot 2R'$ or

$S:S' = abc/2a'b'c'$

Solutions were also offered by Emmett E. Brown, Lima, New York, C. W. Trigg, Los Angeles, George J. Ross, Brooklyn, N. Y., D. G. Burkley, Toronto.

HIGH SCHOOL HONOR ROLL

The editor will be very happy to make special mention of high school classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

For this issue the Honor Roll appears below:

Students of Upper Canada College, Toronto, hold the spot light this month with solutions as follows:

1640. *T. E. Hull.*

1641. *Sidney V. Soanes, David Ker, T. E. Hull, D. G. Burkley, R. W. Laidlaw.*

1643. *T. E. Hull, R. W. L. Laidlaw.*

1645. *T. E. Hull, David Ker, D. G. Burkley, Sidney V. Soanes.*

1644. *Sidney V. Soanes, T. E. Hull, David Ker, D. G. Burkley.*

PROBLEMS FOR SOLUTION

1658. *Proposed by I. N. Warner, Plattesville, Wis.*

By two different methods find the volume of a railway embankment across a valley with the dimensions; width at top 20, at base 45; height 11; length of top 1020; length of base 960.

1659. *Proposed by F. H. Wade, Lewis Institute, Chicago, Ill.*

A tool maker is required to arrange two 8″ by 4″ ellipses with major axes parallel and 2.50″ apart. Find the distance between minor axes if shortest distance from one ellipse to the other is to be exactly 1 inch.

1660. *Proposed by Stephen Droemus, Willard, N. Y.*

Show that every number and its cube when divided by 6 leaves the same remainder.

1661. *Proposed by John Z. Biggerstaff, Portland, Ore.*

$$\frac{x-yz}{\sqrt{(1-y^2)(1-z^2)}}=a$$

$$\frac{y-xz}{\sqrt{(1-x^2)(1-z^2)}}=b$$

$$\frac{z-xv}{\sqrt{(1-x^2)(1-y^2)}}=c.$$

1662. *Proposed by Hugo Brandt, Chicago, Ill.*

In what ratio does the line $x+y=p$ divide the area that the curves $y^2=px$ and $x^2=py$ have in common.

1663. *Proposed by John Meighan, Hillsdale, Mich.*

Prove that there is only one positive integer, n, such that 2^{n+2} is the product of two consecutive even integers.

MATHEMATICS TEACHERS: Use the *Problem Department* for your mental daily dozen.

SCIENCE QUESTIONS

April, 1940

Conducted by Franklin T. Jones

Co-operation is the **process** of **Give and Take.** What can you give? What would you like to take? Please help to make this a real **Department of Co-operation.**

WHAT'S INTERESTING IN SCIENCE ANYWAY!

1. *What is interesting to your students?*
2. *What do you like to teach them?*
3. *Do they like it?*
4. *Are teaching and selling alike? If not, why?*
5. *Do pupils like subjects that make them think?*

GQRA—NEW MEMBERS—April, 1940

Send questions, or answers, or comments to Franklin T. Jones,
10109 Wilbur Avenue, S. E. Cleveland, Ohio

330. *L. C. Hellman, Physical Science Dept., The Bronx High School of Science, New York City.*

331. *Roy Glauber, First-year student in Physics, H.S. of Science, N. Y. C.*

332. *Dr. Matthew T. Jones, Fostoria, Ohio.*

FOR CLASS DISCUSSION

884. *Proposed by L. C. Hellman, Physical Science Dept., High School of Science, The Bronx, New York City. (Elected to the GQRA, No. 330.)*

Here is a question that provoked an interesting and heated discussion in one of my physics classes:

A new electric refrigerator is placed in operation in a well insulated room. The refrigerator is placed in operation with the refrigerator door open.

What happens to the temperature of the room? Why?

885. *Roy Glauber (elected to the GQRA, No. 331), a first year physics student, presented this brain-teaser to his class:*

Given two bars identical in appearance. One is a permanent bar magnet and the other is a bar of soft iron.

Determine which is the bar of iron.

No other materials (not even a pivot) may be used.

WHICH IS HOLLOW?

886. *From Arthur L. Hill, (GQRA, No. 115), Peru, Neb.*

"Here I am with another question for your Department. I keep hoping that soon I may be able to graduate from the lower group which continually proposes questions to the upper group which answers them, and not only for myself but for some of the students in my classes."

Given two metal spheres, same color (perhaps painted so as not to disclose identity), one solid the other with a hollow place in it, presumably

of the shape of a concentric sphere; both weigh exactly the same and both have the same radius but the metals from which each is made is different.

How could one determine of the two which one was hollow?

BURNING STUMPS WITH SALTPETER

887. *The farmers in the country say—*

You can bore a hole in the cut surface of a fresh stump, put in some saltpeter, let the stump stand until it dries out in the spring or early summer, light it and it burns completely clear down into the roots.

Is it true? (Someone please try it.)

How much saltpeter should be used per stump?

Can Chili saltpeter be used just as well?

THE SIMPLE MACHINES ARE OUT OF DATE

888. *Philip B. Sharpe (GQRA, No. 262), Greenwich H.S., Greenwich, N. Y. says—*

"May I use your Department, not to publicize an interesting question but to make a sincere request for help from other GQRA readers?"

Our conventional list of six simple machines seems very faulty to me, and I think that it should be re-arranged and added to.

Accepting the common definition: A SIMPLE MACHINE IS ANY DEVICE WHICH CHANGES THE DIRECTION OR THE RATE OF DOING WORK, I should first like to classify the wedge as a "Driven Inclinded Plane," and the screw as a "Spiral Inclined Plane." Similarly, I should like to call cams and eccentrics "Rotary Inclined Planes," and the wheel and axle, "A Rotary Lever."

This cuts down the classical list to three, but makes several classes of each. The old Three Classes of Levers, I would ignore because almost any simple machine can be used in several manners of application.

The third thing that seems to need doing is to add to this list of three, all the simple machines that have been added to our culture since the time of Aristotle, but have not been added to our Up-to-the-Era science courses. But not being a mechanical engineer, I must ask a great many possibly foolish questions:

What simple machine is a transom opener?

Now that we know enough about aerodynamics to know that sails and airplane wings are not simple inclined planes, what in the world are they?

How about gears, differential pulleys, and different sized pulleys connected by belts? "This one is easy," you will say, but do we agree on them?

When a man slips the clutch on his car, so as to make a steep hill in high gear, he changes the rate at which his engine is working; what simple machine is illustrated here?

And there are the old tricks of pulling sidewise on a taut rope to increase the strain, and of using larger wheels for rough going, as on field gun carriages? We understand the advantages of these things, but just what are they if not simple machines. And if they are simple machines, just which ones are they, pray tell.

And there are large shovels for snow and small ones for iron ore. I know that both shovels are levers, but there is something else there too. If you don't believe it, try shoveling iron ore with a snow shovel. And fill it up!

Last of my suggestions, why in (insert oath to suit your own temperament) isn't a hydraulic press a simple machine?

What other devices have we today that don't fit in very well, under the classical list of six simple machines?

It seems to me that we must consider all these things, and all the others that will occur to *GQRA* readers, to straighten the thing out. This may be starting a big discussion, but isn't it better to admit that we are out-of-date regarding the simple machines, and all help to revise the list?

DO YOU KNOW THE ANSWERS?

Propose questions for this section preferably with short answers. Send in answers. Either will earn membership in the GQRA. Get your pupils to answer.

86. Explain why oxidation is generally reduction. (*C. S. Greenwood, GQRA, No. 135*)
87. Name this peculiar chemical compound—(H I O)Ag. (*C. C. Hall, GQRA, No. 135*)
88. Where were skis invented?
89. From what island does a large share of the nation's manganese come?
90. What mammal lives the longest?

Answers to 76 to 80

76. The shortest interval of time is the interval between the change of the traffic light and the time that the man behind you blows his horn!!!
77. Essential poisons in the diet are arsenic and lead (in extremely minute quantities). So says *Science News Letter* for March 4, 1939.
78. The critical temperature below which growth cannot take place is higher for cancer cells than for normal body cells, so refrigeration is being used to kill cancer cells or prevent their development. (*Science News Letters* for May 13, 1939.)
79. Men grow old before their time for lack of calcium and Vitamin D.
80. Need for the fertility vitamin E increases with age.

OTHER ANSWERS

874. Again. The Calumet & Hecla had immense masses of copper that were in the path of shafts. They were removed, but How?
876. Chamois wiped over a wet glass surface picks up all the water and leaves no droplets behind to form spots. It is absorbent.
877. The Michigan reporter who claimed that the larger auto tags adopted by Michigan would lose large sums of money in gasoline because of extra wind resistance was not a mathematician. The front number on an automobile is usually in front of a fender or the radiator and the eddy currents which form around the number will retard the machine very little even at high speeds. The rear number is now usually located against the trunk and again any eddy currents cannot be serious at any lawful speeds.
879. *Neon Tube Flashing Circuit—Proposed by Lillian A. MacDonald (GQRA, No. 327), South Side H.S., Newark, N. J.* (See School Science and Mathematics, February, 1940, page 193.)

Answer by Dr. Matthew T. Jones (Elected to the GQRA, No. 332), Fostoria, Ohio.

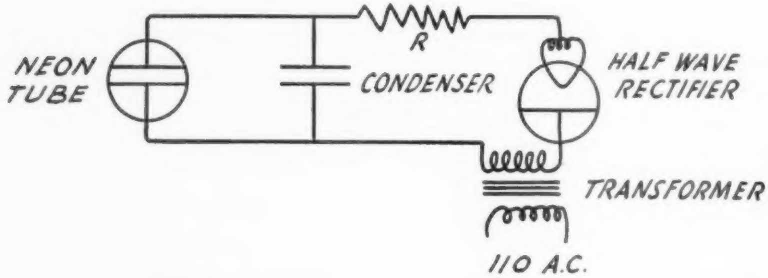
In regard to the neon tube flashing 120 times per second on 60 cycles AC, that is correct, Each plate will glow 60 times a second, alterating to give 120 times visible. One way to make it skip cycles

periodically is to use a condenser to supply the power to make the flash as follows:

The condenser may be a variable capacity made by combining three one-micro-farad units.

The resistance is variable from zero to megohm or higher.

The half-wave rectifier may be a 01-A tube.



If no condenser or resistance, R, the tube will flash 60 times per second. With R, a high resistance (megohm or greater), in the circuit plus a condenser, values adjusted properly, the condenser cannot be charged to a high enough potential to initiate a glow in the neon tube until one or more cycles have passed. (2 cycles will give 30 flashes per second, 3 will give 20, 4 will give 15, etc.)

Thus the rate of discharge of the neon tube can be set at 60 divided by any integer times per second. The transformer as shown in the diagram may not be necessary. If not used, a 40-watt lamp introduced in the circuit between the 110 volt terminal and the half-wave rectifier will control the current.

A similar circuit was supplied to Miss MacDonald by another teacher in New Jersey. She tested out the circuit and found that "the rate of flickering of the neon tube could be controlled very nicely."

Answer by Adrian Struyk (GQRA, No. 75), Clifton H.S., Clifton, N. J.

"This is not a direct answer to No. 879 but may be of interest in connection with it.

"If a hand-driven magneto is available, a neon lamp can be excited by means of it, and made to flash slowly. A quick, short swing of the handle will often produce a single flash on one electrode.

"If the lamp is connected to the 60-cycle supply through a socket at the end of a few feet of lamp cord, the lamp may be shaken rapidly in the direction of the line separating the electrodes. Dark spaces then separate bright semi-circles, showing the glow is not steady. That the patches of light on one side are opposite the dark spaces on the other side shows that the electrodes are excited alternately."

P.S.—For a flasher circuit see Sutton's *Demonstration Experiments in Physics*, page 362, demonstration No. E 263.

JOIN THE GUILD OF QUESTION RAISERS AND ANSWERERS!

Over 330 different Contributors since October, 1934, have been elected members of the GQRA.

BOOKS AND PAMPHLETS RECEIVED

M.K.S. UNITS AND DIMENSIONS, by G. E. M. Jauncey, D.Sc., *Professor of Physics,* and A. S. Lansdorf, M.M.E., *Dean of the School of Engineering Washington University, St. Louis, Mo.* Board. Pages viii+62. 14×21 cm. 1940. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $1.00.

GENERAL PHYSICS FOR STUDENTS OF SCIENCE, by Robert Bruce Lindsay, *Hazard Professor of Physics in Brown University.* Cloth. Pages xiv +534. 14×23 cm. 1940. John Wiley & Sons, Inc., 440 Fourth Avenue, New York, N. Y. Price $3.75.

MATHEMATICAL METHODS IN ENGINEERING, by Theodore v. Karman, *Director of the Guggenheim Aeronautics Laboratory, California Institute of Technology,* and Maurice A. Biot, *Assistant Professor of Mechanics, Columbia University; Honorary Professor, University of Louvain.* Cloth. Pages xii+505. 15×23 cm. 1940. McGraw-Hill Book Co., Inc., 330 W. 42nd St., New York, N. Y. Price $4.00.

THE BOOK OF DIAMONDS, by J. Willard Hershey, M.S., Ph.D., *Department of Chemistry, McPherson, Kansas.* Cloth. Pages xii+142. 15×23.5 cm. 1940. Hearthside Press, The Chemical Publishing Co., Inc., 148 Lafayette Street, New York, N. Y., Price $2.00.

ELEMENTARY COLLEGE MATHEMATICS, by Ernest Lloyd Mackie, Ph.D., *Professor of Mathematics,* and Vinton Asbury Hoyle, Ph.D., *The University of North Carolina, Chapel Hill, North Carolina.* Cloth. Pages ix+331 +78. 14×23 cm. 1940. Ginn and Company, 15 Ashburton Place, Boston, Mass., Price $2.80.

SENIOR MATHEMATICS FOR HIGH SCHOOLS, by Virgil S. Mallory, *Professor of Mathematics and Instructor in the College High School, State Teachers College, Montclair, N. J.,* and Howard F. Fehr, *Assistant Professor of Mathematics and Instructor in the College High School, State Teachers College, Montclair, N. J.* Cloth. Pages ix+442. 12.5×9.5 cm. 1940. Benj. H. Sanborn & Co., 221 E. Twentieth St., Chicago, Ill. Price $1.96.

ELEMENTARY ALGEBRA, by Aaron Freilich, *Chairman, Department of Mathematics, Lafayette High School, New York City,* Henry H. Shanholt, *Chairman, Department of Mathematics, Abraham Lincoln High School, New York City,* Joel S. Georges, *Chairman, Department of Mathematics, Wright Junior College, Chicago.* Cloth. Pages iv+544+25. 13×20 cm. 1940. Silver Burdett Company, 45 East 17th Street, New York, N. Y., Price $1.36.

EVERYDAY ARITHMETIC FOR PRINTERS, by John E. Mansfield, *Head of the Department of Printing, Wentworth Institute, Boston.* Cloth. Pages 3+135. 14×20 cm. McGraw-Hill Book Co., Inc., 330 W. 42nd St., New York, N. Y. Price $1.50.

THE NEW PROGRESS ARITHMETICS, Books A, B, C, D, E, by Philip A. Boyer, *Director, Division of Educational Research, Philadelphia Public Schools,* W. Walker Cheyney, *Elementary Research Supervisor, Philadelphia Public Schools,* Holman White, *Superintendent, District 7, Philadelphia Public Schools.* Paper. Pages each 158 except Book E, which has 190. 20×28 cm. 1940. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price 48 cents each.

REPLICA DIFFRACTION GRATINGS, by R. W. Wood, *Johns Hopkins University*. Paper. 32 pages. 22×28 cm. W. M. Welch Scientific Company, 1515 Sedgwick Street, Chicago, Ill. A catalog of gratings and other apparatus, with a number of experiments described.

CRYSTAL ANALYSIS AND THE POLARIZING MICROSCOPE, by Haller Belt, *Pacific Coast Manager, Bausch & Lomb Optical Company*. An illustrated talk given at the September, 1939, meeting of the California Section of the American Chemical Society. Reprinted from *Pacific Chemical and Metallurgical Industries* September and October 1939. 8 pages. 22×28 cm. Bausch & Lomb Optical Co., Rochester, N. Y.

THE PHOTISMI DE LUMINE OF MAUROLYCUS, Translated from the Latin into English by Henry Crew, *Northwestern University*. Cloth. Pages xix+134. 14.5×21.5 cm. 1940. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $3.00.

MODERN AGRICULTURAL MATHEMATICS, by Maurice Nadler, *Department of Mathematics, Agricultural Division of Newtown High School, Long Island*. Cloth. Pages x+315. 14×21 cm. 1940. Orange Judd Publishing Company, Inc., 15 East 26th Street, New York, N. Y. Price $2.00.

THE PRINCIPLES OF HEREDITY, Second Edition, by Laurence H. Snyder, Sc.D., *Professor of Zoology, Ohio State University*. Cloth. Pages xv+452. 14×22 cm. 1940. D. C. Heath & Company, 285 Columbus Avenue, Boston, Mass. Price $3.50.

REVIEWS AND EXAMINATIONS IN ALGEBRA, by Oswald Tower and Winfield M. Sides, *Department of Mathematics, Phillips Academy, Andover, Mass.* Cloth. Pages v+175. 13×21 cm. D. C. Heath & Company, 285 Columbus Avenue, Boston, Mass. Price $1.20.

BOOK REVIEWS

MATTER, MOTION AND ELECTRICITY, by Henry De Wolf Smyth, *Chairman, Department of Physics, Princeton University*, and Charles Wilbur Ufford, *Professor of Physics, Allegheny College*. Cloth. Pages xiii+648. 15×23 cm. 1939. McGraw-Hill Book Company, Inc., 330 W. 42nd St., New York, N. Y. Price $3.75.

College teachers who are seeking a new approach to physics will find an answer in this text. Students using this text will not say that college physics is just a repetition and extension of the high school course. They are expected to know much physics that is not treated in the text. The book is remarkable for its omissions as well as for what it includes. The molecular interpretation of phenomena is used as the coordinating theory which gives the subject coherence and unity. The first two chapters deal with atoms and molecules and with the phenomena that supply the evidence for the theory of atomic structure. The student is expected to know the mechanics of fluids from his high school work. He should also know trigonometry and understand the calculus notation. Other topics that are not treated in the text but are always included in the conventional course are geometrical optics, much of the subject of sound, thermodynamics, industrial applications of electricity, and fluids in motion. Instead of these the emphasis is on topics essential to an understanding of the developments in modern physics. Molecules, vectors, motion, friction, work and energy, are the topics emphasized in the chapters on mechanics; temperature and heat,

kinetic theory, and change of state, in the section on heat. There is, however, no separation into the conventional major divisions. Static electricity, fields of force, the electronic charge, positive rays, isotopes, radioactivity, spectra, and radiation receive major emphasis. A chapter on rotational motion is introduced when it is needed in the study of the motions of electrons. The construction of such instruments as electric meters, dynamos, and tranformers is entirely omitted or dismissed with a few sentences, but the Van de Graaff generator, the cyclotron, the mass spectrograph, and other instruments of this type are discussed in considerable detail.

One of the very commendable features of the book is the exceptionally clear diction and logical style. The discussion of potential, charge and capacity may be cited as an example. Excellent illustrative problems are solved and explained. The M.K.S. system is used throughout. Use of this book as a class text will require much supplementary aid such as demonstration lectures, problem solving, library work and small group discussion.

G. W. W.

GROWING PLANTS IN NUTRIENT SOLUTION OR SCIENTIFICALLY CONTROLLED GROWTH, by Wayne I. Turner and Victor M. Henry. Cloth. 15×23 cm. xiii+154 pages. 32 illustrations. John Wiley and Sons, Inc. New York, 1939. $3.00.

This is an excellent treatise on the growing of plants without soil. Synonymous terms used to designate this culture are: hydroponics, sand culture, chemiculture, tank farming, water culture, cinder culture and soilless agriculture. This method of plant culture is used commercially by several greenhouse concerns and floral growers. This book gives the details for installation and construction of the equipment need for this kind of culture for both large scale and small scale production, the chemistry and mathematics of the solutions, detailed nutrient formulae for various types of plant crops, methods of controlling growth, detection of imbalance of plant food elements present in the solutions after growth and detection of deficiency symptoms in the plant growth. Index and glossary. A good bibliography of texts, bulletins and periodicals dealing with the subject.

A. G. ZANDER

THE LIVING THOUGHTS OF DARWIN, by Julian Huxley assisted by James Fisher. Cloth. 12.5×18.5 cm. Pages 151. 3 illustrations. Longmans, Green and Co., New York. 1939. $1.00.

This little book is an inexpensive edition portraying the thoughts, ideas and principles enunciated and published by Charles Darwin during his lifetime. The material in this book is taken from these of Darwin's works. The Origin of Species, The Descent of Man, The Formation of Vegetable Mould, Through the Action of Worms, The Fertilization of Orchids, The Variation of Animals and Plants Under Domestication, Autobiography of Darwin.

The book is of value to the interested and thinking layman who obviously would have difficulty in clearing through all of Darwin's works to arrive at the picture portrayed by this volume. The High School Biology teacher might find this book of interest because it gives her an easy approach to the understandings of Darwin's writings.

A. G. ZANDER

BIOLOGY, by Brother H. Charles, F. S. C., Ph.D. (Charles F. Severin), *Professor of Biology at St. Mary's College, Winona, Minn.* 14×21 cm.

Cloth. Pages vii+408. Illustrations 220. Bruce Publishing Co., Milwaukee, Wis. 1939. $1.72.

A well laid out text, easily understood written primarily for Catholic High Schools. The illustrations are particularly correctly and well labelled. The legends are meaningful and refer to material in the text. Too often we find the relation between illustrations and the context of a biology book weak, here this has been overcome. Of particular interest is the chapter called Variation and Heredity, in which among other things, the various theories of evolution are understandingly explained as is the status of this subject today. The organization is on the basis of the interrelations of plant, animal and man as regards their various life functions. There are 33 chapters, a good glossary and index. The book could in the opinion of the reviewer have more material on animals.

A. G. ZANDER

Learning Activities in Elementary Algebra, by Earl J. Burnett, M.A., Head of Department of Mathematics, Monroe High School, Rochester, N. Y., and Regina Grosswege, M.A. Department of Mathematics, Elwood High School, Elwood, Indiana. Edited by Halbert C. Christofferson, Ph.D., Professor of Mathematics and Director of Secondary Education, Miami University, Oxford, Ohio. Paper. Pages iv+228 (Mastery tests to accompany—20 pages bound separately). 20×26.5 cm. 1939. College Entrance Book Company, Inc., 104 Fifth Avenue, New York City. Price 66 cents.

Learning Activities in Plane Geometry, by Earl J. Burnett, M.A., Head of Department of Mathematics, Monroe High School, Rochester, N. Y., and William E. Batzler, M.A., Department of Mathematics, Battle Creek High School, Battle Creek, Michigan. Edited by Halbert C. Christofferson, Ph.D. Paper. Pages viii+248 (Mastery tests to accompany—30 pages bound separately). 20×26.5 cm. 1939. College Entrance Book Company, Inc., 104 Fifth Avenue, New York City. Price 68 cents.

These two workbooks provide many pictures and articles which attempt to motivate the study of mathematics by presentation of historical developments and modern applications. A supply of starred exercises offers a challenge to the brighter pupils. The authors state that ample working space is provided for all examples. In common with many workbooks, this statement is true if the student uses the correct method with no extra steps; if he struggles along with some false moves he will be crowded in many places. Reviews are provided at appropriate places, with mastery tests bound in a separate pamphlet although the reason for separate binding is not made clear. Both books contain four place values of the natural trigonometric functions, the geometry workbook contains in addition a table of squares and square roots. The typography is excellent, the glossy paper produced an annoying reflection when working under electric lights.

CECIL B. READ
University of Wichita

The World of Insects, by Carl D. Duncan, Professor of Entomology and Botany, San Jose State College, and Gayle Pickwell, Professor of Zoology, San Jose State College. Cloth. Pages ix+409. 15×23 cm. 1939. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York, N. Y. Price $3.50.

This book is remarkable for its many excellent illustrations—nearly 200—all of which are new. More than one-half of the illustrations are

photographs of living insects in their natural habitats. There are numerous well executed drawings showing details of insect anatomy.

The book has been written with the beginner in mind: the style is clear and simple and technical words are defined as they are introduced. Unusual interest is given to the discussion by the emphasis placed on the activities of insects. In the authors' words, "Insects alive and active overbalance insects dead and pinned." The emphasis is apparent from an inspection of the chapter headings: 1. Introduction. 2. Insect Structures. 3. How Insects Grow Up. 4. The Growing Up of a Swallowtail Butterfly. 5. Insect Foods and Feeding Habits. 6. Some Insect Food-getting Devices. 7. How Insects Reproduce Themselves. 8. How Insects Get Air. 9. How Insects Move. 10. How Insects are Protected. 11. Insect Voices. 12. Insect Fitness. 13. Insect Orders. 14. Social Life among the Insects. 15. The Value of Insects. 16. Injurious Insects and their Control. 17. Where to Look for Insects. 18. Rearing Insects. 19. How to Collect and Preserve Insects.

The last five chapters contain a great deal of practical information, so that the book is really a handbook and text combined. There is a three page list of references and a well organized index. A high grade glazed paper has been used. The binding is of the substantial sort that one has learned to expect from this publisher. This book is recommended as a reference work for general students in high school and college and as a delightful bit of reading for anyone interested in the biology of insects.

EDWARD C. COLIN
Chicago Teachers College

Felix Klein. Elementary Mathematics from an Advanced Standpoint. Geometry. Translated from the third German edition by E. R. Hedrick, Vice President and Provost, The University of California and C. A. Noble, Professor of Mathematics, Emeritus, The University of California. Cloth. Pages ix+214. 16×23.5 cm. 1939. The Macmillan Company, New York, N. Y. Price $3.50.

Those of us who have enjoyed the English translation of the first volume of Klein's three volume work will appreciate the appearance of this translation of the second volume. The course of Klein's lectures which is reproduced had as a purpose a survey of the entire field of geometry although material from all fields of mathematics is utilized.

The content of the book is divided into three parts: The Simplest Geometric Manifolds; Geometric Transformations; Systematic Discussion of Geometry and Its Foundations. Throughout the book there are many references to the historical development of various concepts. If one considers the book from the point of view of the teacher, it is obvious that its purpose is to aid in the arrangement and classification of geometric facts acquired in various scattered courses—not to offer the basis for a new beginning course. There are many places where concepts from various fields are brought to bear to aid in the precise formulation of geometric facts. It would be hard to find a better presentation of the various aspects which geometry may take.

Advanced students will no doubt benefit greatly from a reading of portions of the book, the third part should be required reading for anyone planning to teach high school geometry and will be found interesting and valuable to the experienced teacher even if the first two parts may seem somewhat difficult. Klein has stated the case very well when he calls attention to the fact that, for example, non-euclidean geometry is one of the few parts of mathematics which any teacher may be asked about at any moment and that the teacher of mathematics who could give no answer

would make as poor an impression as the teacher of physics who is unable to say anything about radium.

The appearance of the book and the typography are excellent, only two trivial errors were noted (pages 176, 181).

CECIL B. READ
University of Wichita

NEW AND REVISED TEXTBOOKS

MODERN PHYSICS, by Charles E. Dull, *Head of Science Department, West Side High School, and Supervisor of Science for the Junior and Senior High Schools, Newark, New Jersey.* Revised. Cloth. Pages x+587+xxv. 15×23.5 cm. 1939. Henry Holt and Company, 257 Fourth Avenue, New York, N. Y. Price $1.80.

This book, a favorite in many high schools since it first appeared in 1934, has now been so thoroughly revised and improved that it is in reality a new book. It is sound physics, very interesting and attractive, and filled with helps for teacher and pupil.

HOUSEHOLD PHYSICS, Third Edition, by Walter G. Whitman, *State Teachers College, Salem, Mass.* Cloth. Pages vi+436. 14.5×23 cm. 1939. John Wiley & Sons, Inc., 440 Fourth Avenue, New York, N. Y. Price $3.00.

This is a physics text for girls by a master teacher. For fifteen years its previous editions have been popular. This edition insures it a further period of usefulness.

OUTLINE OF PHYSIOLOGY, by William R. Amberson, *Professor of Physiology, University of Maryland,* and Dietrich C. Smith, *Associate Professor of Physiology, University of Maryland.* Cloth. Pages vii+412. 17.5×25.5 cm. 1939. F. S. Crofts and Company, 41 Union Square, West, New York, N. Y. Price $4.00.

This book presents the textual material for a brief course for beginning college students. It contains both historical and modern developments and includes the modern concepts of physics and chemistry necessary for an understanding of the course.

DISCOVERING OUR WORLD, by Wilbur L. Beauchamp, Mary Melrose, and Glenn O. Blough. Book Three. Cloth. 464 pages. 12.5×19 cm. 1939. Scott, Foresman and Company, 623 S. Wabash Avenue, Chicago, Ill. Price $1.00.

The third book in this elementary science series by three outstanding leaders in the field is a splendid example of what can be done to make science both attractive and useful to children. The beautiful photographs, many of them in color, the vivid descriptions, and the complete explanations will develop in the pupils of the middle grades an admiration for science and a desire to continue its study.

MECHANICS, by L. Raymond Smith, *Instructor in Industrial Physics, William L. Dickinson High School, Jersey City, N. J.* Revised Edition. Cloth. Pages xiv+299. 12×19 cm. 1939. McGraw-Hill Book Company, Inc., 330 W. 42nd Street, New York, N. Y. Price $1.75.

This is a revision of the text originally published in 1922. The chief points of difference are some new diagrams and improved diction. Answer books are available for teachers.

RHODIUM METAL USED FOR TELESCOPE MIRRORS

Rhodium metal, deposited on glass by evaporation, makes rugged reflecting surfaces for telescope mirrors which withstand chemical corrosion, Dr. W. W. Coblentz of the National Bureau of Standards in Washington told the American Astronomical Society.

The rhodium mirror surfaces seem especially desirable for small mirrors used by amateur astronomers in their instruments. The rhodium, evaporated to a vapor in a vacuum and then condensed on the glass surface, avoids the chemical corrosive destruction which occurs on some aluminum coated mirrors in telescopes. Aluminization, developed and applied in observatories on the West Coast, works well in that climate. But since first introduced a few years ago it has been discovered that elsewhere the mirror surfaces corrode and must be replaced.

Dr. Coblentz has found that rhodium applied by evaporation has better reflecting properties than the same metal applied by electrolysis. Rhodium, of course, is one of the most resistant of all metals to chemical action. In fact this property will probably preclude the use of rhodium on large telescope mirrors unless it is applied over a silver coating or chromium. Rhodium is itself so resistant to chemical action that it is virtually impossible to remove it by chemicals once it is in place. Only when an underlying, more soluble film is first placed on the mirror can it be stripped off. Dr. Coblentz foresees the use of rhodium for the amateur astronomer's field where small mirrors, easily handled, are used.

EDUCATION IN THE SOUTHERN MOUNTAINS

Comparatively few of the children in the most mountainous counties of Georgia, Kentucky, or Virginia, attend high school, unless they go to non-public schools or to public schools outside of their home counties. A survey of education in the Southern Mountains made by the U. S. Office of Education reveals this fact. Opportunities for secondary education frequently are not available in the home counties of high-school-age children, or if available, the distances are great and transportation is not provided, it was learned. While fairly high percentages of those 16 to 20 years of age attend school, many are still in the grades. They are either marking time because there are no high schools to which they can go, or else they are still in the grades because of retardation, the Office of Education points out in its survey report, "Education in the Southern Mountains."

METAL DECLARED TO EXIST BENEATH ARIZONA METEOR CRATER

Confidence that metal exists beneath famous meteor crater in Arizona was expressed by D. Moreau Barringer, Jr., long associated with the mining company that owns this site, commenting on recent reports that indicated all the metal contained in the meteorite spattered outside when it hit.

"The last three drill holes which were sunk all encountered quantities of meteoric material at depths of around 500 to 700 feet below the bottom of the crater," said Mr. Barringer. "Each of the three holes was stopped by 'boulders' of meteoric material enclosed in a matrix of crushed sandstone. Since the largest bit that we were using only had a diameter of 6⅝ inches, we were unable to penetrate the main mass of the fragments. However, the holes served to corroborate with surprising exactness both the geological deductions and the results of the geophysical surveys."